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Dynamics of intersecting brane systems — Classification and their applications

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Abstract: We present dynamical intersecting brane solutions in higher-dimensional gravitational theory coupled to dilaton and several forms. Assuming the forms of metric, form fields, and dilaton field, we give a complete classification of dynamical intersecting brane solutions with/without M-waves and Kaluza-Klein monopoles in eleven-dimensional supergravity. We apply these solutions to cosmology and black holes. It is shown that these give FRW cosmological solutions and in some cases Lorentz invariance is broken in our world. If we regard the bulk space as our universe, we may interpret them as black holes in the expanding universe. We also discuss lower-dimensional effective theories and point out naive effective theories may give us some solutions which are inconsistent with the higher-dimensional Einstein equations.

Keywords: p-branes, Black Holes in String Theory, Intersecting branes models, M-Theory

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1 Introduction

Recently there have been works on dynamical spacetime-dependent solutions of supergravity involving branes which are of cosmological interest. The dynamical solutions of supergravity have a number of important applications. In the original version [1], one considers a spacetime-dependent brane solution with five-form flux and gravity in the ten-dimensional type IIB supergravity. In the presence of the spacetime dependence in the background metric, one finds, even for the general black p-brane system [2–4], that the

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structure of warp factor which depends on the space and time is different from the usual "product type" ansatz [5–8].

In addition to spacetime-dependent brane solutions in higher-dimensional supergravities, there are several analyses of lower-dimensional effective theories after compactifying the internal space [9–12]. The same considerations also apply to the string theories which are of much interest as an approach to behavior of the early universe. However, it has been pointed out that the four-dimensional effective theories for warped compactification of ten-dimensional type IIB supergravity may not provide solutions in the original higher-dimensional theories [9, 13]. This caution can be generalized in various p-brane solutions [14] and intersecting branes as shown in this paper. This possible inconsistency has been known for some time, but it should be kept in mind in the recent study of moduli in higher-dimensional theories using four-dimensional effective theories.

Another significant fact is that more general dynamical brane solutions arise if the gravity is coupled not only to single gauge field but to several combinations of scalars and forms as intersecting brane solutions in the supergravity. The intersecting brane solutions were originally found by Güven in eleven-dimensional supergravity [15]. After that, many authors investigated related solutions such as intersecting membranes, and they constructed static new solutions of intersecting branes [16–25]. For a nice review, see [26]. Furthermore, a different class of dynamical brane solutions which depend on both time and space coordinates have been found in [27], and special intersecting dynamical solutions of D4-D8 are given in [14].

In the present paper, we give general dynamical solutions of intersecting brane systems in D-dimensional theories, which may have more general applications to cosmology and black hole physics, and discuss their implications to lower-dimensional effective theories. We show that these solutions give FRW universe if we regard the homogeneous and isotropic part of the brane world-volumes as our spacetime, whereas they give black hole solutions in FRW universe if we regard the bulk transverse space as our spacetime. We also show that in the former case, Lorentz invariance may appear broken in our four-dimensional world though more elaborate solutions may be necessary to obtain realistic models. Although our solutions contain general intersecting brane solutions including D-branes and NS-branes, we discuss M-branes mainly in our following discussions for simplicity. Other branes can be obtained by dimensional reductions and T-duality.

The paper is organised as follows. In section 2, we first consider intersecting p-brane systems in D-dimensions and derive general dynamical intersecting brane solutions under certain metric ansätze. In section 3, focusing on intersecting M-brane systems in the eleven-dimensional supergravity, we give a classification of dynamical intersecting brane solutions without M-wave and Kaluza-Klein(KK)-monopole, and discuss spacetime structure of the intersecting branes. A complete classification of these solutions is given in appendix B. In section 4, applications of these solutions to cosmology and black hole physics are discussed. In section 5, dynamical intersecting brane solutions involving M-wave and KK-monopole are discussed together with their applications to cosmology and black holes. In section 6, we discuss lower-dimensional effective theories for the warped compactification of the brane systems in eleven-dimensional supergravity and discuss that Lorentz invariance

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in our spacetime may appear broken in our solutions. Section 7 is devoted to concluding remarks. Dynamical solutions of single branes are summarized in appendix A, and the complete classification of intersecting M-branes are given in appendix B. Solutions with M-wave and KK-monopole are given in appendix C, and their intersections with M-branes are given in appendix D.

2 Solutions of dynamical intersecting branes

In this section, we consider dynamical intersecting brane systems in D dimensions. We write down the Einstein equations under certain metric ansätze, which are a generalization of those of known static intersecting p-brane solutions. We then solve the Einstein equations and present the solutions explicitly. To compare the results of intersecting p-brane with those of single p-branes, we summarize the dynamical solutions of single p-branes in appendix A.

Let us consider a gravitational theory with the metric g_{MN}, dilaton ϕ, and anti-symmetric tensor fields of rank (p_I+2), where I denotes the type of the corresponding branes. The action for the intersecting-brane system is written as

$$S = \frac{1}{2\kappa^2}\int\left[R\ast\mathbf{1}_D - \frac{1}{2}d\phi\wedge\ast d\phi - \sum_I \frac{1}{2(p_I+2)!}e^{c_I\phi}F_{(p_I+2)}\wedge\ast F_{(p_I+2)}\right], \tag{2.1}$$

where κ^2 is the D-dimensional gravitational constant, $\ast$ is the Hodge dual operator in the D-dimensional spacetime, c_I is a constant given by

$$c_I^2 = 4 - \frac{2(p_I+1)(D-p_I-3)}{D-2}. \tag{2.2}$$

The expectation values of fermionic fields are assumed to be zero. The action (2.1) describes the bosonic part of $D=11$ or $D=10$ supergravities; we simply drop ϕ and put $c_I=0$ and $p_I=2$ for $D=11$, whereas we set $c_I=-1$ for the NS-NS 3-form and $c_I=\frac{1}{2}(3-p_I)$ for forms coming from the R-R sector for $D=10$. There may be Chern-Simons terms in the action, but they are irrelevant in our following search for the orthogonally intersecting solutions. To describe more general supergravities in lower dimensions, we should include several scalars as in refs. [2, 28], but for simplicity we disregard this complication in this paper.

The field equations are given by

$$\begin{aligned}
R_{MN} &= \frac{1}{2}\partial_M\phi\partial_N\phi + \frac{1}{2}\sum_I\frac{1}{(p_I+2)!}e^{\epsilon_I c_I\phi}\\
&\quad\times\left[(p_I+2)F_{MA_2\cdots A_{p_I+2}}F_N{}^{A_2\cdots A_{p_I+2}} - \frac{p_I+1}{D-2}g_{MN}F^2_{(p_I+2)}\right], &(2.3a)\\
\Box\phi &= \frac{1}{2}\sum_I\frac{\epsilon_I c_I}{(p_I+2)!}e^{c_I\phi}F^2_{(p_I+2)}, &(2.3b)\\
d\left(e^{c_I\phi}\ast F_{(p_I+2)}\right) &= 0, &(2.3c)
\end{aligned}$$

where $\Box$ is the D-dimensional D'Alembertian.

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To solve the field equations, we assume the D-dimensional metric of the form

$$ds^2 = \mathcal{A}(t,z)u_{ij}(z)dz^i dz^j - \mathcal{B}(t,z)dt^2 + \sum_{\alpha=1}^{p} \mathcal{C}^{(\alpha)}(t,z)(dx^\alpha)^2, \tag{2.4}$$

where $u_{ij}(z)$ is the metric of the $(D-p-1)$-dimensional Z space which depends only on the $(D-p-1)$-dimensional coordinates z^i. $\mathcal{A}$, $\mathcal{B}$ and $\mathcal{C}^{(\alpha)}$ are given by

$$\mathcal{A} = \prod_I [h_I(t,z)]^{a_I}, \quad \mathcal{B} = \prod_I [h_I(t,z)]^{b_I}, \quad \mathcal{C}^{(\alpha)} = \prod_I [h_I(t,z)]^{c_I^{(\alpha)}}. \tag{2.5}$$

where the parameters a_I, b_I and $c_I^{(\alpha)}$ are defined by

$$a_I = \frac{p_I+1}{D-2}, \qquad b_I = -\frac{D-p_I-3}{D-2}, \qquad c_I^{(\alpha)} = \begin{cases} b_I & \text{for} \quad \alpha \in I \\ a_I & \text{for} \quad \alpha \notin I \end{cases}, \tag{2.6}$$

and $h_I(t,z)$, which depends on t and z^i, is a straightforward generalization of the harmonic function associated with a brane I in a static brane system [16].

We also assume that the scalar field ϕ and the gauge field strength $F_{(p+2)}$ are given by

$$e^\phi = \prod_I h_I^{\epsilon_I c_I/2}, \quad F_{(p_I+2)} = d(h_I^{-1}) \wedge \Omega(\mathrm{X}_I), \tag{2.7}$$

where X_I is the space associated with a brane I, and ϵ_I is defined by

$$\epsilon_I = \begin{cases} + & \text{for the electric brane} \\ - & \text{for the magnetic brane} \end{cases}, \tag{2.8}$$

and $\Omega(\mathrm{X}_I) = dt \wedge dx^{p_1} \wedge \cdots \wedge dx^{p_I}$ is the volume (p_I+1)-form. The field strength in (2.7) is written for electric ansatz, but the final results are basically the same for magnetic ansatz. In what follows, we write our formulae mainly for electric case with comments on modifications for magnetic case.

Let us assume [16]

$$\begin{aligned} \mathcal{A}^{(D-p-3)}\,\mathcal{B} \prod_{\alpha=1}^{p} \mathcal{C}^{(\alpha)} &= 1\,, \\ \mathcal{B}^{-1} \prod_{\alpha\in I} \left(\mathcal{C}^{(\alpha)}\right)^{-1} e^{\epsilon_I c_I \phi} &= h_I^2. \end{aligned} \tag{2.9}$$

The Einstein equations (2.3a) then reduce to

$$\frac{1}{2}\sum_{I,I'}\left[M_{II'}-2\delta_{II'}-2\left(a_I-\delta_{II'}a_{I'}\right)\right]\partial_t\ln h_I\partial_t\ln h_{I'}$$
$$+\sum_I(2a_I-b_I)h_I^{-1}\partial_t^2h_I-2\prod_I h_I^{-1}\sum_{I'}b_{I'}h_{I'}^{-1}\triangle_{\rm Z}h_{I'}=0, \tag{2.10a}$$

$$2\sum_I h_I^{-1}\partial_t\partial_i h_I+\sum_{I,I'}\left(M_{II'}-2\delta_{II'}\right)\partial_t\ln h_I\partial_i\ln h_{I'}=0, \tag{2.10b}$$

$$\prod_{J'}h_{J'}^{-b_{J'}}\sum_\gamma\prod_J h_J^{c_J^{(\gamma)}}\sum_I\left[c_I^{(\gamma)}h_I^{-1}\partial_t^2h_I-\left(c_I^{(\gamma)}\partial_t\ln h_I-\sum_{I'}c_{I'}^{(\gamma)}\partial_t\ln h_{I'}\right)\partial_t\ln h_I\right]$$
$$-\prod_{J'}h_{J'}^{-a_{J'}}\sum_\gamma\prod_J h_J^{c_J^{(\gamma)}}\sum_I c_I^{(\gamma)}h_I^{-1}\triangle_{\rm Z}h_I=0, \tag{2.10c}$$

$$R_{ij}({\rm Z})+\frac{D-2}{2}u_{ij}\prod_J h_J\sum_I\left[a_Ih_I^{-1}\partial_t^2h_I+\left\{a_I\partial_t\ln h_I-\sum_{I'}a_{I'}\partial_t\ln h_{I'}\right\}\partial_t\ln h_I\right]$$
$$-\frac{1}{2}u_{ij}\sum_I h_I^{-1}a_I\triangle_{\rm Z}h_I-\frac{1}{4}\sum_{I,I'}\left(M_{II'}-2\delta_{II'}\right)\partial_i\ln h_I\partial_j\ln h_{I'}=0, \tag{2.10d}$$

where $R_{ij}({\rm Z})$ is the Ricci tensor of the metric u_{ij}, and $M_{II'}$ is given by

$$M_{II'}\equiv b_Ib_{I'}+\sum_\alpha c_I^{(\alpha)}c_{I'}^{(\alpha)}+(D-p-3)a_Ia_{I'}+\frac{1}{2}\epsilon_I\epsilon_{I'}c_Ic_{I'}\,. \tag{2.11}$$

Let us consider eq. (2.10b). We can rewrite this as

$$\sum_{I,I'}\left[M_{II'}+2\delta_{II'}\frac{\partial_t\partial_i\ln h_I}{\partial_t\ln h_I\partial_i\ln h_I}\right]\partial_t\ln h_I\partial_i\ln h_{I'}=0. \tag{2.12}$$

In order to satisfy this equation for arbitrary coordinate values and independent functions h_I, the second term in the square bracket must be constant:

$$\frac{\partial_t\partial_i\ln h_I}{\partial_t\ln h_I\partial_i\ln h_I}=k_I\,. \tag{2.13}$$

Then in order for (2.12) to be satisfied identically, we must have

$$M_{II'}+2k_I\delta_{II'}=0. \tag{2.14}$$

Using eqs. (2.2), (2.6) and (2.11), we get

$$\begin{aligned}M_{II}&=(p_I+1)b_I^2+(p-p_I)a_I^2+(D-p-3)a_I^2+\frac{1}{2}c_I^2\\&=2.\end{aligned} \tag{2.15}$$

This means that the constant k_I in eq. (2.14) is $k_I=-1$, namely

$$M_{II'}=2\delta_{II'}. \tag{2.16}$$

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It then follows from eq. (2.13) that

$$\partial_i \partial_t [h_I(t,z)] = 0\,. \tag{2.17}$$

As a result, the warp factor h_I must be separable as

$$h_I(t,z) = K_I(t) + H_I(z)\,. \tag{2.18}$$

For $I \neq I'$, eq. (2.16) gives the intersection rule on the dimension $\bar{p}$ of the intersection for each pair of branes I and I' ($\bar{p} \leq p_I, p_{I'}$) [16, 19, 29, 30]:

$$\bar{p} = \frac{(p_I+1)(p_{I'}+1)}{D-2} - 1 - \frac{1}{2}\epsilon_I c_I \epsilon_{I'} c_{I'}. \tag{2.19}$$

Let us next consider the gauge field. Under the ansatz (2.7) for electric background, we find

$$dF_{(p_I+2)} = h_I^{-1}(2\partial_i \ln h_I \partial_j \ln h_I + h_I^{-1}\partial_i\partial_j h_I) dz^i \wedge dz^j \wedge \Omega(\mathrm{X}_I) = 0. \tag{2.20}$$

Thus, the Bianchi identity is automatically satisfied. Also the equation of motion for the gauge field becomes

$$\begin{aligned} d\left[e^{-c_I\phi} * F_{(p_I+2)}\right] &= -d\left[\partial_i h_I \left\{*_\mathrm{Z} dy^i \wedge *_\mathrm{X}\Omega(\mathrm{X}_I)\right\}\right] \\ &= -\left(\partial_t\partial_i h_I dt + \triangle_\mathrm{Z} h_I dy^i\right) \wedge \left[*_\mathrm{Z} dy^i \wedge *_\mathrm{X}\Omega(\mathrm{X}_I)\right] = 0, \end{aligned} \tag{2.21}$$

where $*_\mathrm{X}$, $*_\mathrm{Z}$ denotes the Hodge dual operator on $\mathrm{X}(\equiv \cup_I X_I)$ and Z, respectively, and we have used eqs. (2.9). Hence we again find the condition (2.18) and

$$\triangle_\mathrm{Z} h_I = 0. \tag{2.22}$$

We note that the roles of the Bianchi identity and field equations are interchanged for magnetic ansatz [16, 19], but the net result is the same.

Let us finally consider the scalar field equation. Substituting the scalar field and the gauge field in (2.7), and the warp factor (2.18) into the equation of motion for the scalar field (2.3b), we obtain

$$\begin{aligned} -\prod_{I''} h_{I''}^{-b_{I''}} \sum_I \epsilon_I c_I \left[h_I^{-1}\partial_t^2 K_I + \partial_t \ln h_I \sum_{I'} \partial_t \ln h_{I'} - (\partial_t \ln h_I)^2\right] \\ + \prod_{I''} h_{I''}^{-a_{I''}} \sum_I h_I^{-1} \epsilon_I c_I \triangle_\mathrm{Z} H_I = 0. \end{aligned} \tag{2.23}$$

This equation is satisfied if

$$\partial_t^2 K_I = 0, \tag{2.24a}$$

$$\triangle_\mathrm{Z} H_I = 0, \tag{2.24b}$$

$$\sum_I \epsilon_I c_I \left[\partial_t \ln h_I \sum_{I'} \partial_t \ln h_{I'} - (\partial_t \ln h_I)^2\right] = 0. \tag{2.24c}$$

eq. (2.24a) gives $K_I = A_I t + B_I$, where where A_I and B_I are integration constants. Eq. (2.24c) can be satisfied *only if there is only one function h_I depending on both z^i and t, which we denote with the subscript $\tilde{I}$, and other functions are either dependent on z^i or constant.* Hence we have

$$\begin{aligned} K_{\tilde{I}} &= A_{\tilde{I}} t + B_{\tilde{I}}, \\ K_I &= B_I, \quad (I \neq \tilde{I}). \end{aligned} \tag{2.25}$$

The remaining Einstein equations (2.10) now reduce to

$$\sum_{I,I'} \left[a_I - \delta_{II'} a_{I'}\right] \partial_t \ln h_I \partial_t \ln h_{I'} = 0, \tag{2.26a}$$

$$\sum_I \left[\partial_t \ln h_I \sum_{I'} \partial_t \ln h_{I'} - (\partial_t \ln h_I)^2\right] = 0, \tag{2.26b}$$

$$R_{ij}(\mathrm{Z}) = 0. \tag{2.26c}$$

Obviously the first two sets of equations (2.26a) and (2.26b) are automatically satisfied by our solutions in which there is only one function $h_{\tilde{I}}$ depending on both t and z^i. Given the set of solutions to eqs. (2.18), (2.24b), (2.25), and (2.26c), we have thus obtained general intersecting dynamical brane solutions (2.4). For static (time-independent) case, our solutions are consistent with the harmonic function rule [31], but are more general with spacetime-dependent functions. Note that the internal space is not warped [9] if the function H_I is trivial.

As a special example, we consider the case

$$u_{ij} = \delta_{ij} \,, \tag{2.27}$$

where δ_{ij} is the $(D-p-1)$-dimensional Euclidean metric. In this case, the solution for H_I can be obtained explicitly as

$$H_I(z) = 1 + \sum_k \frac{Q_{I,k}}{|\boldsymbol{z} - \boldsymbol{z}_k|^{D-p-3}}, \tag{2.28}$$

where $Q_{I,k}$'s are constant parameters and $\boldsymbol{z}_k$ represent the positions of the branes in Z space.[1] For $K_{\tilde{I}} = 0$ ($A_{\tilde{I}} = B_{\tilde{I}} = 0$), the metric describes the known static and extremal multi-black hole solution with black hole charges $Q_{I,k}$ [16, 19, 32].

3 Classification of dynamical intersecting M-branes

Now, we give a classification of multiple intersections of M-branes in eleven dimensions. The intersections of D-branes and other branes can be obtained by dimensional reductions and T-duality. We look for the possible configurations of intersecting branes by use of (2.19). It turns out that no configuration is possible for more than eight branes [33]. In what follows, we present explicit solutions. The case with M-waves or KK-monopoles will be discussed later (section 5).

[1] Here we show the solution without compactification of Z space. One may also discuss the case that q-dimensions of Z space are smeared, which gives the different power of harmonics, i.e. $|\boldsymbol{z} - \boldsymbol{z}_k|^{-(D-p-3-q)}$ $(q \leq D-p-2)$.

	0	1	2	3	4	5	6	7	8	9	10	$\tilde{I}$
M5	○	○	○	○	○	○						$\surd$
M5	○	○	○	○			○	○				
	t	x^1	x^2	x^3	x^4	x^5	y^6	y^7	z^1	z^2	z^3	

Table 1. M5-M5 brane system

3.1 Dynamical intersecting M-branes

In our solutions (2.4), only one time-dependent brane is allowed, which we denote by $\tilde{I}$. Then we have

$$h_{\tilde{I}} = h_{\tilde{I}}(t,z) \equiv A_{\tilde{I}}\, t + H_{\tilde{I}}(z)\,. \tag{3.1}$$

Here we set $B_{\tilde{I}} = 0$ without loss of generality. We write the solution as

$$ds^2 = \mathcal{A}(t,z)\left[-g_0(t,z)dt^2 + \sum_{\tilde{\alpha}\in\tilde{I}} g_{\tilde{\alpha}}(t,z)(dx^{\tilde{\alpha}})^2 + \sum_{\alpha\notin\tilde{I}} g_\alpha(z)(dy^\alpha)^2 + u_{ij}(z)dz^i dz^j\right], \tag{3.2}$$

with

$$\begin{aligned} \mathcal{A} &= \left[A_{\tilde{I}}t + H_{\tilde{I}}(z)\right]^{a_{\tilde{I}}} \prod_{I\neq\tilde{I}} H_I(z)^{a_I} \;, \quad g_0 = \left[A_{\tilde{I}}t + H_{\tilde{I}}(z)\right]^{-1} \prod_{I\neq\tilde{I}} H_I(z)^{-1}, \\ g_{\tilde{\alpha}} &= \left[A_{\tilde{I}}t + H_{\tilde{I}}(z)\right]^{-1} \prod_{I\neq\tilde{I}} H_I^{-\gamma_I^{(\tilde{\alpha})}} \;, \quad g_\alpha = \prod_{I\neq\tilde{I}} H_I^{-\gamma_I^{(\alpha)}}\,, \end{aligned} \tag{3.3}$$

where

$$a_{\tilde{I}} = \frac{p_{\tilde{I}}+1}{D-2} \;, \quad \text{and} \quad \gamma_I^{(\alpha)} = \begin{cases} 1 & \text{for} \quad \alpha\in I \\ 0 & \text{for} \quad \alpha\notin I \end{cases}\,. \tag{3.4}$$

Here we divide the coordinates of brane world-volume ($\{x^\alpha\}$) into two parts ($\{x^{\tilde{\alpha}}\}, \{y^\alpha\}$): the first are the $p_{\tilde{I}}$-dimensional coordinates $x^{\tilde{\alpha}}$ which describe the time-dependent brane $\tilde{I}$, and the second are the $(p-p_{\tilde{I}})$-dimensional coordinates y^α which represent the remaining space of the brane world-volume.

Let us now give one simple example of M5-M5 brane system. The intersection rule (2.19) gives the brane configuration in table 1. The mark $\surd$ in the table shows which brane is time dependent, though in this case there is no difference whichever of the two M5's is chosen. The metric is then given by

$$\begin{aligned} ds^2 = (h_{\tilde{5}}H_5)^{2/3}\Bigg[&(h_{\tilde{5}}H_5)^{-1}\left(-dt^2 + \sum_{\tilde{\alpha}=1}^{3}(dx^{\tilde{\alpha}})^2\right) + h_{\tilde{5}}^{-1}\sum_{\tilde{\alpha}=4}^{5}(dx^{\tilde{\alpha}})^2 \\ &+H_5^{-1}\sum_{\alpha=6}^{7}(dy^\alpha)^2 + u_{ij}dz^i dz^j\Bigg]\,, \end{aligned} \tag{3.5}$$

that is

$$\begin{aligned}
&\mathcal{A} = (h_{\tilde{5}} H_5)^{2/3}\,, \\
&g_0 = g_{\tilde{1}} = g_{\tilde{2}} = g_{\tilde{3}} = (h_{\tilde{5}} H_5)^{-1} \;\;,\;\; g_{\tilde{4}} = g_{\tilde{5}} = h_{\tilde{5}}^{-1}, \\
&g_1 = g_2 = H_5^{-1},
\end{aligned} \tag{3.6}$$

where

$$h_{\tilde{5}} = A_{\tilde{5}} t + H_{\tilde{5}}(z)\,. \tag{3.7}$$

The form field is given by

$$F_{(4)} = -\, *_{\mathrm{Z}} (dh_{\tilde{5}}) \wedge dx^{\tilde{4}} \wedge dx^{\tilde{5}} - *_{\mathrm{Z}} (dH_5) \wedge dy^6 \wedge dy^7\,, \tag{3.8}$$

where $*_{\mathrm{Z}}$ is the Hodge dual operator in the three-dimensional Z space.

The complete classification and explicit metrics for intersecting brane systems are summarized in appendix B.

3.2 Spacetime structure of the intersecting branes

Near branes ($|\boldsymbol{z}| \sim 0$), the spacetime structure is the same as that of the static solution unless the dimension of Z space is one. This is because the metric components diverge as $|\boldsymbol{z}| \to 0$ and the static harmonic parts dominate the time-dependent terms. In that case, we know that M2-M2, M2-M5, M2-M2-M2, M5-M5-M5, M2-M2-M5-M5 systems are regular on the branes.

If Z space is one-dimensional, then we have $h_{\tilde{I}} = A_{\tilde{I}} t + \sum_k Q_{I,k} |z - z_k|$. Hence any points on the branes ($z = z_k$) are regular, and time dependent.

Even if the near-brane structure is regular, we expect another type of singularity may appear at $h_{\tilde{I}}(t, z) = 0$. Since $h_{\tilde{I}}$ is a linear function of t, it vanishes once for any position $\boldsymbol{z}$ at $t = -H_{\tilde{I}}(z)/A_{\tilde{I}}$.

When we take the limit of $H_{\tilde{I}} \to 0$ (or finite) as $|\boldsymbol{z}| \to \infty$ for $\dim(\mathrm{Z}) > 1$ (or $|z|$ is finite for $\dim(\mathrm{Z}) = 1$), the spacetime turns out to be time dependent and homogeneous. To see its dynamical behaviour, we introduce a new time coordinate

$$\tau = \tau_0 (A_{\tilde{I}} t)^{(a_{\tilde{I}}+1)/2}, \tag{3.9}$$

where $\tau_0 = \frac{2}{A_{\tilde{I}}(a_{\tilde{I}}+1)}$. The asymptotic solution is rewritten as

$$ds^2 = -d\tau^2 + \left(\frac{\tau}{\tau_0}\right)^{2q_{\tilde{I}}} \sum_{\tilde{\alpha}} (dx^{\tilde{\alpha}})^2 + \left(\frac{\tau}{\tau_0}\right)^{2q_{\tilde{X}}} \left(\sum_{\alpha} (dy^{\alpha})^2 + u_{ij} dz^i dz^j\right)\,, \tag{3.10}$$

where

$$q_{\tilde{I}} = \frac{a_{\tilde{I}} - 1}{a_{\tilde{I}} + 1} = -\frac{D - p_{\tilde{I}} - 3}{D + p_{\tilde{I}} - 1}\,, \quad q_{\tilde{X}} = \frac{a_{\tilde{I}}}{a_{\tilde{I}} + 1} = \frac{p_{\tilde{I}} + 1}{D + p_{\tilde{I}} - 1}. \tag{3.11}$$

More explicitly, for the case of M-theory ($D = 11$), we find

$$a_{\tilde{I}} = 1/3,\ q_{\tilde{I}} = -1/2,\ q_{\tilde{\cancel{I}}} = 1/4, \quad \text{for } \tilde{I} = \text{M2 } (p_{\tilde{I}} = 2)\,, \tag{3.12a}$$

$$a_{\tilde{I}} = 2/3,\ q_{\tilde{I}} = -1/5,\ q_{\tilde{\cancel{I}}} = 2/5, \quad \text{for } \tilde{I} = \text{M5 } (p_{\tilde{I}} = 5)\,. \tag{3.12b}$$

Hence, we find a Kasner-like expansion:

$$p_{\tilde{I}}\, q_{\tilde{I}} + p_{\tilde{\cancel{I}}}\, q_{\tilde{\cancel{I}}} = 1\,, \tag{3.13a}$$

$$p_{\tilde{I}}(q_{\tilde{I}})^2 + p_{\tilde{\cancel{I}}}(q_{\tilde{\cancel{I}}})^2 = 1\,. \tag{3.13b}$$

where $p_{\tilde{\cancel{I}}} = (D - p_{\tilde{I}} - 1)$ is the dimension of the space volume perpendicular to the $\tilde{I}$-brane world-volume. Eq. (3.13a) is always satisfied for any brane configuration, but eq. (3.13b) is true only for M-theory because no dilaton appears.

This time dependence is also correct if we fix the position in Z space, although the metric is locally inhomogeneous in the bulk space.

4 Applications to cosmology and black holes

4.1 Cosmology

Now we discuss how these solutions are applied to our physical world. Since we consider time-dependent solutions, it is natural to discuss cosmology. Suppose that our three-dimensional universe is a part of branes. Since our universe is isotropic and homogeneous, same branes must contain this whole three dimensions. Hence we should look for whether there is a solution with an isotropic and homogeneous three space from a list of our solutions given in appendix B. Note that this does not mean that the three space must be contained in all branes. We find just six cases, i.e., M2-M5, M5-M5, M5-M5-M5, M2-M5-M5, M2-M2-M5, and M2-M2-M5-M5 brane systems. In some cases, we have two different expansion laws for our universe depending on whether the brane on which our world exists is time dependent or not.

We then compactify some dimensions to fit our three space. We assume that our universe is one of the branes (or its three-dimensional part), which can be the time-dependent one ($\tilde{I}$) or the static one ($I(\neq \tilde{I})$). Hence our universe stays at a constant position in the bulk space ($\boldsymbol{z} = \boldsymbol{z}_k$). Note that among the above spacetimes, only M2-M5 and M2-M2-M5-M5 brane systems are regular on the branes. For other configurations, the curvature diverges there. Hence one need invoke a mechanism to avoid singularity if our world is confined on the brane.

We describe our three space Ξ by the coordinates $\boldsymbol{\xi} = (\xi^1, \xi^2, \xi^3)$. There are two possibilities: One is that Ξ belongs to some part of the time-dependent brane world-volume $\mathrm{X}_{\tilde{I}}$ (case 1), and the other is that Ξ is contained in a part of only static brane world-volume $\mathrm{Y}_I (I \neq \tilde{I})$, which does not belong to $\mathrm{X}_{\tilde{I}}$ (case 2).

For the case 1, the metric (3.2) is described by

$$ds^2 = ds_4^2 + ds_{p-3}^2 + ds_{\text{bulk}}^2\,, \tag{4.1}$$

where

$$
\begin{aligned}
ds_4^2 &= \mathcal{A}\left[-g_0 dt^2 + g_\xi \sum_{\tilde\alpha\in\Xi}(dx^{\tilde\alpha})^2\right], \\
ds_{p-3}^2 &= \mathcal{A}\left[\sum_{\tilde\alpha\notin\Xi,\,\tilde\alpha\in\tilde I} g_{\tilde\alpha}(dx^{\tilde\alpha})^2 + \sum_{\alpha\notin\tilde I} g_\alpha (dy^\alpha)^2\right], \\
ds_{\text{bulk}}^2 &= \mathcal{A}\, u_{ij} dz^i dz^j\,.
\end{aligned}
\tag{4.2}
$$

From our ansatz, $g_{\tilde\alpha}$'s for our three space ($\tilde\alpha \in \Xi$) are the same, which we denote g_ξ. ds_{p-3}^2 is the part of compactified brane world-volume, and ds_{bulk}^2 describes the empty bulk space.

We have to describe our 4-dimensional universe in the Einstein frame, which is given by

$$
\begin{aligned}
d\bar s_4^2 &\equiv \prod_{\tilde\alpha\notin\Xi,\,\tilde\alpha\in\tilde I}(\mathcal{A} g_{\tilde\alpha})^{1/2}\prod_{\alpha\notin\tilde I}(\mathcal{A} g_\alpha)^{1/2}\, ds_4^2 \\
&= h_{\tilde I}^{s_{\tilde I}}(t,z) F_{\tilde I}(z)\left[-f_0(z)dt^2 + f_\xi(z) d\boldsymbol{\xi}^2\right],
\end{aligned}
\tag{4.3}
$$

where

$$
\begin{aligned}
s_{\tilde I} &= \frac{1}{2}\left[-(p_{\tilde I}-1) + \frac{(p-1)}{(D-2)}(p_{\tilde I}+1)\right], \\
F_{\tilde I} &= \prod_{I\neq\tilde I} H_I^{\frac{(p_I+1)(p-1)}{2(D-2)}} \times \prod_{\tilde\alpha\notin\Xi,\,\tilde\alpha\in\tilde I}\left(\prod_{I\neq\tilde I} H_I^{-\gamma_I^{(\tilde\alpha)}/2}\right) \times \prod_{\alpha\notin\tilde I}\left(\prod_{I\neq\tilde I} H_I^{-\gamma_I^{(\alpha)}/2}\right), \\
f_0 &= \prod_{I\neq\tilde I} H_I^{-1}\,, \quad f_\xi = \prod_{I\neq\tilde I} H_I^{-\gamma_I^{(\xi)}}\,.
\end{aligned}
\tag{4.4}
$$

Here note that the middle factor in $F_{\tilde I}$ has the exponent $\gamma_I^{(\tilde\alpha)}$ which is nonvanishing for the case where the coordinate $x^{\tilde\alpha}$ belongs to time-dependent brane as well as time-independent I brane.

For the case 2, we have

$$
\begin{aligned}
ds_4^2 &= \mathcal{A}\left[-g_0 dt^2 + g_\xi \sum_{\alpha\in\Xi,\notin\tilde I}(dy^\alpha)^2\right], \\
ds_{p-3}^2 &= \mathcal{A}\left[\sum_{\tilde\alpha\in\tilde I} g_{\tilde I}(dx^{\tilde\alpha})^2 + \sum_{\alpha\notin\Xi,\tilde I} g_\alpha (dy^\alpha)^2\right], \\
ds_{\text{bulk}}^2 &= \mathcal{A} u_{ij} dz^i dz^j\,.
\end{aligned}
\tag{4.5}
$$

Hence the 4-dimensional metric of our universe in the Einstein frame is

$$
\begin{aligned}
d\bar s_4^2 &\equiv \prod_{\tilde\alpha\in\tilde I}(\mathcal{A} g_{\tilde\alpha})^{1/2}\prod_{\alpha\notin\Xi,\notin\tilde I}(\mathcal{A} g_\alpha)^{1/2}\, ds_4^2 \\
&= h_{\tilde I}^{s_{\tilde I}}(t,z) F_{\tilde I}(z)\left[-f_0(z)dt^2 + h_{\tilde I}(t,z) f_\xi(z) d\boldsymbol{\xi}^2\right],
\end{aligned}
\tag{4.6}
$$

where

$$
\begin{aligned}
s_{\tilde{\not{I}}} &= \frac{1}{2}\left[-(p_{\tilde{I}}+2)+\frac{(p-1)}{(D-2)}(p_{\tilde{I}}+1)\right], \\
F_{\tilde{\not{I}}} &= \prod_{I\neq\tilde{I}} H_I^{\frac{(p_{\tilde{I}}+1)(p-1)}{2(D-2)}} \times \prod_{\tilde{\alpha}\in\tilde{I}}\left(\prod_{I\neq\tilde{I}} H_I^{-\gamma_I^{(\tilde{\alpha})}/2}\right) \times \prod_{\alpha\notin\Xi,\notin\tilde{I}}\left(\prod_{I\neq\tilde{I}} H_I^{-\gamma_I^{(\alpha)}/2}\right).
\end{aligned}
\tag{4.7}
$$

Since we fix our universe at some position in the bulk Z space, $\boldsymbol{z}$ is constant in the above metric. Hence we find the isotropic and homogeneous universe. We introduce the cosmic time τ, which is defined by

$$
\tau = \begin{cases} \tau_{\tilde{I}}(A_{\tilde{I}}t)^{(s_{\tilde{I}}+2)/2} & \text{for the case 1} \\ \tau_{\tilde{\not{I}}}(A_{\tilde{I}}t)^{(s_{\tilde{\not{I}}}+2)/2} & \text{for the case 2} \end{cases}, \tag{4.8}
$$

where $\tau_{\tilde{I}} = 2/[A_{\tilde{I}}(s_{\tilde{I}}+2)]$ and $\tau_{\tilde{\not{I}}} = 2/[A_{\tilde{I}}(s_{\tilde{\not{I}}}+2)]$, respectively. The scale factor of the universe is given by

$$
\begin{aligned}
a_{\tilde{I}} &= (A_{\tilde{I}}t)^{s_{\tilde{I}}/2} = \left(\frac{\tau}{\tau_0}\right)^{\beta_{\tilde{I}}}, \\
a_{\tilde{\not{I}}} &= (A_{\tilde{I}}t)^{(s_{\tilde{\not{I}}}+1)/2} = \left(\frac{\tau}{\tau_0}\right)^{\beta_{\tilde{\not{I}}}},
\end{aligned}
\tag{4.9}
$$

where

$$
\beta_{\tilde{I}} = \frac{s_{\tilde{I}}}{(s_{\tilde{I}}+2)}\ , \text{ and }\quad \beta_{\tilde{\not{I}}} = \frac{(s_{\tilde{\not{I}}}+1)}{(s_{\tilde{\not{I}}}+2)}. \tag{4.10}
$$

The power of the cosmological solution for each possible model is listed in table 2. Since the time dependence in the metric comes from only one M-brane (or D-brane) in the intersections, the obtained expansion law may be too simple. In fact, we find the Minkowski space, which is static, in almost every case.

In order to find an expanding universe, one may have to smear and compactify the vacuum bulk space as well as the brane world-volume. Suppose k-dimensions of the bulk Z space are smeared and compactified, where $k < \dim(\mathrm{Z}) = D-p-1$. The metric in the Einstein frame is multiplied by the extra factor $\mathcal{A}^{k/2}$. As a result, we find new exponents of the metric are

$$
\begin{aligned}
s_{\tilde{I}}^{(k)} &= s_{\tilde{I}} + \frac{k(p_{\tilde{I}}+1)}{2(D-2)}, \\
s_{\tilde{\not{I}}}^{(k)} &= s_{\tilde{\not{I}}} + \frac{k(p_{\tilde{I}}+1)}{2(D-2)}.
\end{aligned}
\tag{4.11}
$$

The power of the scale factor is given by the same equations (4.10) by replacing $s_{\tilde{I}}$ with $s_{\tilde{I}}^{(k)}$ (or $s_{\tilde{\not{I}}}$ with $s_{\tilde{\not{I}}}^{(k)}$). We also show these explicit powers in table 2. However, even for the

	branes	dim(Z)	$s_{\tilde{I}}$ or $s_{\tilde{X}}$	$\beta_{\tilde{I}}$ or $\beta_{\tilde{X}}$	$\beta^{(1)}_{\tilde{I}}$ or $\beta^{(1)}_{\tilde{X}}$	$\beta^{(2)}_{\tilde{I}}$ or $\beta^{(2)}_{\tilde{X}}$	$\beta^{(3)}_{\tilde{I}}$ or $\beta^{(3)}_{\tilde{X}}$
case 1 ($\tilde{I}$ =M5)	M2-M5	4	$-1/3$	$-1/5$	0	1/7	1/4
	M5-M5	3	0	0	1/7	1/4	–
	M5-M5-M5	1	2/3	1/4	–	–	–
	M2-M5-M5	3	0	0	1/7	1/4	–
	M2-M2-M5	3	0	0	1/7	1/4	–
	M2-M2-M5-M5	3	0	0	1/7	1/4	–
case 2 ($\tilde{I}$ =M2)	M2-M5	4	$-7/6$	$-1/5$	0	1/7	1/4
	M2-M5-M5	3	-1	0	1/7	1/4	–
	M2-M2-M5	3	-1	0	1/7	1/4	–
	M2-M2-M5-M5	3	-1	0	1/7	1/4	–

Table 2. The power exponent $\beta_{\tilde{I}}$ (or $\beta_{\tilde{X}}$) of the scale factor $a_{\tilde{I}}$ (or $a_{\tilde{X}}$) of possible 4-dimensional cosmological model is given, i.e. $a \propto \tau^{\beta}$, where τ is the cosmic time. The last three columns are for the case of smeared and compactified bulk space.

fastest expanding case $a \propto \tau^{1/4}$, the power is too small to give a realistic expansion law such as that in the matter dominated era ($a \propto \tau^{2/3}$) or that in the radiation dominated era ($a \propto \tau^{1/2}$).

Hence we conclude that in order to find a realistic expansion of the universe in this type of models, one have to include additional "matter" fields on the brane.

4.2 Time-dependent black holes

Since the static (or stationary) intersecting brane system describes the microstate of a black hole, it may be natural to apply the present solutions to a time-dependent spacetime with a black hole. In this case, just as the case of a static black hole, we should compactify all brane world-volume, and obtain the d-dimensional spacetime, where $d \equiv D - p = \dim(\mathrm{Z}) + 1$. Our metric is described as

$$ds^2 = ds_d^2 + ds_p^2 \,, \tag{4.12}$$

where

$$\begin{aligned} ds_d^2 &= \mathcal{A}\left[-g_0 dt^2 + u_{ij} dz^i dz^j\right] , \\ ds_p^2 &= \mathcal{A}\left[\sum_{\tilde{\alpha}\in\tilde{I}} g_{\tilde{\alpha}} (dx^{\tilde{\alpha}})^2 + \sum_{\alpha\notin\tilde{I}} g_{\alpha} (dy^{\alpha})^2\right] . \end{aligned} \tag{4.13}$$

The compactification of ds_p^2 gives the effective d-dimensional spacetime, whose metric in the Einstein frame is given by

$$d\bar{s}_d^2 = \prod_{\tilde{\alpha}\in\tilde{I}} (\mathcal{A} g_{\tilde{\alpha}})^{1/(d-2)} \prod_{\alpha\notin\tilde{I}} (\mathcal{A} g_{\alpha})^{1/(d-2)} \, \mathcal{A}\left(-g_0 dt^2 + u_{ij} dz^i dz^j\right) , \tag{4.14}$$

which is rewritten explicitly as

$$d\bar{s}_d^2 = h_{\tilde{I}}^{s_{\rm BH}} F_{\rm BH}(z)\left(-f_0(z) dt^2 + h_{\tilde{I}}(t,z) u_{ij} dz^i dz^j\right) , \tag{4.15}$$

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where

$$s_{\rm BH} = -\frac{d-3}{d-2},$$

$$F_{\rm BH}(z) = \prod_{I\neq\tilde{I}} H_I^{\frac{(p_I+1)(p+2)}{(d-2)(D-2)}} \prod_{\tilde{\alpha}\in\tilde{I}} \left(\prod_{I\neq\tilde{I}} H_I^{-\gamma_I^{(\tilde{\alpha})}/(d-2)}\right) \prod_{\alpha\notin\tilde{I}} \left(\prod_{I\neq\tilde{I}} H_I^{-\gamma_I^{(\alpha)}/(d-2)}\right) . \quad (4.16)$$

We look for a four or higher dimensional "black hole", i.e. $d \geq 4$, or equivalently $\dim(\mathrm{Z}) \equiv D-p-1 \geq 3$. In M-theory, this implies that $p \leq 7$. The corresponding brane systems are M2-M2, M2-M5, M5-M5, M5-M5-M5, M2-M5-M5, and M2-M2-M5-M5.

The near-brane geometry is the same as the static one because $h_{\tilde{I}} \to H_{\tilde{I}}(z)$ as $\boldsymbol{z} \to \boldsymbol{z}_k$ and then the geometry approaches the static solution. If it has a horizon geometry, we can regard the present time-dependent solution as a black hole. We know that only two cases (M2-M2-M2, M2-M2-M5-M5) give regular black hole spacetimes in the static limit.

On the other hand, the asymptotic structure is completely different. The static solution has an asymptotically flat geometry, but the present solution is time dependent. In fact, setting $h_{\tilde{I}} = t/t_0 + H_{\tilde{I}}$, from eq. (4.15) in the limit of $|\boldsymbol{z}| \to \infty$, we find

$$\begin{aligned} d\bar{s}_d^2 &= \left(\frac{t}{t_0}\right)^{s_{\rm BH}} \left[-dt^2 + \left(\frac{t}{t_0}\right) u_{ij} dz^i dz^j\right] \\ &= -d\tau^2 + a_{\rm BH}^2(\tau)\, u_{ij} dz^i dz^j \,, \end{aligned} \quad (4.17)$$

where

$$a_{\rm BH} = \left(\frac{\tau}{\tau_0}\right)^{\beta_{\rm BH}} , \quad (4.18)$$

with

$$\beta_{\rm BH} = \frac{s_{\rm BH}+1}{s_{\rm BH}+2} = \frac{1}{d-1} \ , \quad \tau_0 = \frac{2}{s_{\rm BH}+2}\, t_0 = \frac{2(d-2)}{d-1} t_0 \,. \quad (4.19)$$

Hence our solution approaches asymptotically the FRW universe with the scale factor $a_{\rm BH}$. So, if the static solution gives a black hole, then we can regard the present solution as a black hole in the expanding universe. In table 3, we show a list of the power exponent of asymptotic expanding universe for the possible black hole (or black object) model.

If we smear and compactify the vacuum bulk Z space just as the case of cosmology, we find the different power exponent of the scale factor, which is also shown in table 3. As a result, we always find the same power $\beta_{\rm BH} = 1/(d-1)$ for a d-dimensional black hole (or black object). This power exponent is obtained for the universe filled by stiff matter whose equation of state is $P = \rho$. Therefore we may regard the present d-dimensional solution as a time-dependent black hole in the stiff-matter dominated universe.

Here we give one explicit example of M2-M2-M5-M5 brane system. We assume that one M2 brane is time-dependent.

$$d\bar{s}_4^2 = -\left(h_{\tilde{2}} H_2 H_5 H_{5'}\right)^{-1/2} dt^2 + \left(h_{\tilde{2}} H_2 H_5 H_{5'}\right)^{1/2} \left(dr^2 + r^2 d\Omega_2^2\right) , \quad (4.20)$$

branes	d	$\tilde{I}$	$s_{\rm BH}$	$\beta_{\rm BH}$	$\beta_{\rm BH}^{(k)}$	BH
M2-M2	7	M2	$-4/5$	$1/6$	$1/5,\ 1/4,\ 1/3\ (k=1,2,3)$	
M2-M5	5	M2	$-2/3$	$1/4$	$1/3\ (k=1)$	
		M5	$-2/3$	$1/4$	$1/3\ (k=1)$	
M5-M5	4	M5	$-1/2$	$1/3$	–	
M5-M5-M5	4	M5	$-1/2$	$1/3$	–	
M2-M5-M5	4	M2	$-1/2$	$1/3$	–	
		M5	$-1/2$	$1/3$	–	
M2-M2-M5	4	M2	$-1/2$	$1/3$	–	
		M5	$-1/2$	$1/3$	–	
M2-M2-M2	5	M2	$-2/3$	$1/4$	$1/3\ (k=1)$	$\surd$
M2-M2-M5-M5	4	M2	$-1/2$	$1/3$	–	$\surd$
		M5	$-1/2$	$1/3$	–	$\surd$

Table 3. The power exponent of the asymptotic expansion for "BH" spacetime. Only the brane systems marked in the column "BH" have regular horizons.

where

$$\begin{aligned} h_{\tilde{2}} &= \frac{t}{t_0} + \frac{Q_{\tilde{2}}}{r}, \\ H_2 &= 1 + \frac{Q_2}{r}, \quad H_5 = 1 + \frac{Q_5}{r}, \quad H_{5'} = 1 + \frac{Q_{5'}}{r}, \end{aligned} \tag{4.21}$$

This metric is rewritten as

$$d\bar{s}_4^2 = -(\tilde{H}_{\tilde{2}} H_2 H_5 H_{5'})^{-1/2} d\tau^2 + a_{\rm BH}^2(\tau)(\tilde{H}_{\tilde{2}} H_2 H_5 H_{5'})^{1/2} \left(dr^2 + r^2 d\Omega_2^2\right), \tag{4.22}$$

where

$$\tilde{H}_{\tilde{2}} = 1 + \frac{\tilde{Q}_{\tilde{2}}(\tau)}{r}, \quad \text{and} \quad a_{\rm BH} = \left(\frac{\tau}{\tau_0}\right)^{\frac{1}{3}}, \tag{4.23}$$

with

$$\tilde{Q}_{\tilde{2}} \equiv \left(\frac{\tau}{\tau_0}\right)^{-\frac{4}{3}} Q_{\tilde{2}}, \quad \text{and} \quad \tau_0 \equiv \frac{4}{3}\, t_0\,. \tag{4.24}$$

The power 1/3 in eq. (4.23) is the one given in table 3.

5 Intersecting M-branes with M-waves and KK-monopoles

Now we discuss the dynamical intersecting brane solutions including M-waves and KK-monopoles in eleven dimensions. The dimensional reduction of these generates the Kaluza-Klein electric or magnetic charges in the 2-form field strengths [32, 34–36]. In $(D-1)$-dimensional spacetime, one can obtain the electric 0-brane and the magnetic $(D-5)$-brane solutions. Lifting up those solutions by one dimension, we obtain the KK-wave and KK-monopole in D-dimensions, respectively. In particular, KK-wave is called "M-wave" in eleven-dimensional theory [37, 38]. We briefly summarize those objects in appendix C.

We extend our brane solutions given in section 3 to the cases with M-waves and/or KK-monopoles. For the static case, there is a classification of the multiple intersecting branes with the M-waves and/or KK-monopoles [33, 34].

		0	1	2	3	4	5	6	7	8	9	10	$\tilde{I}$	cos	BH
M2-W	M2	$\circ$	$\circ$	$\circ$									(a)	–	$\surd$
	W	$\circ$	ζ										(b)	–	$\surd$
M5-W	M5	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$						(a)	$\surd$	$\surd$
	W	$\circ$	ζ										(b)	$\surd$	$\surd$
M2-KKM	M2	$\circ$	$\circ$	$\circ$									(a)	$\surd$	$\surd$
	KKM	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	ζ	$\mathcal{A}_8^{(\mathrm{m})}$	$\mathcal{A}_9^{(\mathrm{m})}$	$\mathcal{A}_{10}^{(\mathrm{m})}$	(b)	$\surd$	$\surd$
	M2	$\circ$							$\circ$			$\circ$	(c)	$\surd$	–
	KKM	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	ζ	$\mathcal{A}_8^{(\mathrm{m})}$	$\mathcal{A}_9^{(\mathrm{m})}$	$\mathcal{A}_{10}^{(\mathrm{m})}$	(d)	$\surd$	–
M5-KKM	M5	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$						(a)	$\surd$	$\surd$
	KKM	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	ζ	$\mathcal{A}_8^{(\mathrm{m})}$	$\mathcal{A}_9^{(\mathrm{m})}$	$\mathcal{A}_{10}^{(\mathrm{m})}$	(b)	$\surd$	$\surd$
	M5	$\circ$	$\circ$	$\circ$	$\circ$				$\circ$			$\circ$	(c)-1,2	$\surd$	–
	KKM	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	ζ	$\mathcal{A}_8^{(\mathrm{m})}$	$\mathcal{A}_9^{(\mathrm{m})}$	$\mathcal{A}_{10}^{(\mathrm{m})}$	(d)-1,2	$\surd$	–
W-KKM	W	$\circ$	ζ^1										(a)	$\surd$	$\surd$
	KKM	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	ζ^7	$\mathcal{A}_8^{(\mathrm{m})}$	$\mathcal{A}_9^{(\mathrm{m})}$	$\mathcal{A}_{10}^{(\mathrm{m})}$	(b)	$\surd$	$\surd$
KKM-KKM	KKM	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\mathcal{B}_5^{(\mathrm{m})}$	$\mathcal{B}_6^{(\mathrm{m})}$	ζ	$\circ$	$\circ$	$\mathcal{B}_{10}^{(\mathrm{m})}$	(a)	$\surd$	–
	KKM	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	ζ	$\mathcal{A}_8^{(\mathrm{m})}$	$\mathcal{A}_9^{(\mathrm{m})}$	$\mathcal{A}_{10}^{(\mathrm{m})}$			
	KKM	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	ζ^5	$\mathcal{B}_6^{(\mathrm{m})}$	$\circ$	$\circ$	$\mathcal{B}_9^{(\mathrm{m})}$	$\mathcal{B}_{10}^{(\mathrm{m})}$	(b)	$\surd$	–
	KKM	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	ζ^7	$\mathcal{A}_8^{(\mathrm{m})}$	$\mathcal{A}_9^{(\mathrm{m})}$	$\mathcal{A}_{10}^{(\mathrm{m})}$			

Table 4. The brane configurations following the intersection rule with M-wave (W) and/or KK-monopole (KKM). The brane systems marked in the columns "cos" and "BH" can be used for cosmological and black hole systems. The labelling (a), (b), $\cdots$ in the column "$\tilde{I}$" denotes which brane (or M-wave, KK-monopole) is time dependent. In the second case of M5-KKM system, there are two possibilities which space dimensions can be our three space, i.e., the case 1: $[(\xi^1, \xi^2, \xi^3) = (x^1, x^2, x^3)]$ and the case 2: $[(\xi^1, \xi^2, \xi^3) = (x^4, x^5, x^6)]$. We show them by (c)-1 (c)-2, or (d)-1, (d)-2.

We first show the intersection rule for the branes with M-wave and/or KK-monopoles, which is summarized in table 4. In the table, circles indicate where the brane world-volumes enter, ζ represents the coordinate of the KK-monopole, and the time-dependent branes are indicated by (a) and (b) for different solutions. When the solutions can be used for cosmology and black hole physics, they are marked in the corresponding columns.

There are two configurations for two KK-monopole system as shown in table 4. The metric of the former and the latter cases are given by

$$ds^2{}_{\mathrm{2KKM}} = -dt^2 + \sum_{\alpha=1}^{4}(dx^\alpha)^2 + h_{\mathrm{m}1}\sum_{\alpha=5}^{6}(dz^\alpha)^2 + h_{\mathrm{m}2}\sum_{\alpha=8}^{9}(dz^\alpha)^2 + h_{\mathrm{m}1}h_{\mathrm{m}2}\left(dz^{10}\right)^2$$
$$+(h_{\mathrm{m}1}h_{\mathrm{m}2})^{-1}\left[d\zeta + \mathcal{B}_5^{(\mathrm{m})}dz^5 + \mathcal{B}_6^{(\mathrm{m})}dz^6 + \mathcal{A}_8^{(\mathrm{m})}dz^8 + \mathcal{A}_9^{(\mathrm{m})}dz^9 + \left(\mathcal{A}_{10}^{(\mathrm{m})} + \mathcal{B}_{10}^{(\mathrm{m})}\right)dz^{10}\right]^2 . \tag{5.1}$$

$$ds^2{}_{\mathrm{2KKM}} = -dt^2 + \sum_{\alpha=1}^{4}(dx^\alpha)^2 + h_{\mathrm{m}2}\left(dz^6\right)^2 + h_{\mathrm{m}1}\left(dz^8\right)^2 + h_{\mathrm{m}1}h_{\mathrm{m}2}\sum_{\alpha=9}^{10}(dz^\alpha)^2$$

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$$
\begin{aligned}
&+(h_{\rm m2})^{-1}\left(d\zeta^5+\mathcal{B}_6^{(\rm m)}dz^6+\mathcal{B}_9^{(\rm m)}dz^9+\mathcal{B}_{10}^{(\rm m)}dz^{10}\right)^2\\
&+(h_{\rm m1})^{-1}\left(d\zeta^7+\mathcal{A}_8^{(\rm m)}dz^8+\mathcal{A}_9^{(\rm m)}dz^9+\mathcal{A}_{10}^{(\rm m)}dz^{10}\right)^2 .
\end{aligned}\tag{5.2}
$$

Next, we present the brane systems with M-wave or one KK-monopole. As we mentioned, only one brane can have time dependence in the present approach. It is also true for the warp factor from M-wave or KK-monopole. Hence we have two cases for time-dependent solutions, i.e. we can have either one time-dependent brane or time-dependent M-wave (or KK-monopole).

In the former case, the metric forms for the spacetimes with M-wave and with KK-monopole, respectively, are written as

$$
\begin{aligned}
ds_{\rm W}^2 &= \mathcal{A}(t,z)\Big[g_0(t,z)\left\{-dt^2+(d\zeta^{\tilde{1}})^2+f_{\rm w}(z)(dt-d\zeta^{\tilde{1}})^2\right\}\\
&\quad+\sum_{\tilde{\alpha}\neq 1,\tilde{\alpha}\in\tilde{I}} g_{\tilde{\alpha}}(t,z)(dx^{\tilde{\alpha}})^2+\sum_{\alpha\notin\tilde{I}}g_\alpha(z)(dy^\alpha)^2+u_{ij}(z)dz^idz^j\Big],
\end{aligned}\tag{5.3}
$$

$$
\begin{aligned}
ds_{\rm KKM}^2 &= \mathcal{A}(t,z)\Big[-g_0(t,z)dt^2+\sum_{\tilde{\alpha}\in\tilde{I}}g_{\tilde{\alpha}}(t,z)(dx^{\tilde{\alpha}})^2\\
&\quad+\sum_{\alpha\notin\tilde{I}}g_\alpha(z)(dy^\alpha)^2+h_{\rm m}(z)u_{ij}dz^idz^j+h_{\rm m}^{-1}(z)\left(d\zeta+\mathcal{A}_i^{(\rm m)}(z)dz^i\right)^2\Big],
\end{aligned}\tag{5.4}
$$

with

$$
\begin{aligned}
\mathcal{A} &= \left[A_{\tilde{I}}t+H_{\tilde{I}}(z)\right]^{a_{\tilde{I}}}\prod_{I\neq\tilde{I}}H_I(z)^{a_I}\ , \quad g_0 = \left[A_{\tilde{I}}t+H_{\tilde{I}}(z)\right]^{-1}\prod_{I\neq\tilde{I}}H_I(z)^{-1},\\
g_{\tilde{\alpha}} &= \left[A_{\tilde{I}}t+H_{\tilde{I}}(z)\right]^{-1}\prod_{I\neq\tilde{I}}H_I^{-\gamma_I^{(\tilde{\alpha})}}\ , \quad g_\alpha = \prod_{I\neq\tilde{I}}H_I^{-\gamma_I^{(\alpha)}} .
\end{aligned}\tag{5.5}
$$

where

$$
a_{\tilde{I}}=\frac{p_{\tilde{I}}+1}{D-2}\ , \quad \text{and}\quad \gamma_I^{(\alpha)}=\begin{cases}1 & \text{for}\ \ \alpha\in I\\ 0 & \text{for}\ \ \alpha\notin I\,.\end{cases}\tag{5.6}
$$

The coordinate ζ belongs to either X or Z. In the KK-monopole case, dim(Z)=3.

If M-wave or KK-monopole depends on time, we find the following solutions:

$$
\begin{aligned}
ds_{\rm W}^2 &= \mathcal{A}(z)\left[g_0(z)\left\{-dt^2+\left(d\zeta^{\tilde{1}}\right)^2+f_{\rm w}(t,z)\left(dt-d\zeta^{\tilde{1}}\right)^2\right\}\right.\\
&\qquad\left.+\sum_\alpha g_\alpha(z)(dx^\alpha)^2+u_{ij}(z)dz^idz^j\right],
\end{aligned}\tag{5.7}
$$

$$
\begin{aligned}
ds_{\rm KKM}^2 &= \mathcal{A}(z)\left[-g_0(z)dt^2+\sum_\alpha g_\alpha(z)(dx^\alpha)^2\right.\\
&\qquad\left.+h_{\rm m}(t,z)u_{ij}dz^idz^j+h_{\rm m}^{-1}(t,z)\left(d\zeta+\mathcal{A}_i^{(\rm m)}(z)dz^i\right)^2\right],
\end{aligned}\tag{5.8}
$$

	0	1	2	3	4	5	6	7	8	9	10	$\tilde{I}$	cos	BH
M2	$\circ$	$\circ$	$\circ$									(a)	$\surd$	$\surd$
M5	$\circ$	$\circ$		$\circ$	$\circ$	$\circ$	$\circ$					(b)	$\surd$	$\surd$
W	$\circ$	$\circ$										(c)	$\surd$	$\surd$
	t	$\zeta^{\tilde{1}}$	y^2	$x^{\tilde{3}}$	$x^{\tilde{4}}$	$x^{\tilde{5}}$	$x^{\tilde{6}}$	z^1	z^2	z^3	z^4			

Table 5. M2-M5-W brane system

with

$$\mathcal{A} = \prod_I H_I^{a_I}\,, \quad g_0 = \prod_I H_I^{-1}\,, \quad g_\alpha = \prod_I H_I^{-\gamma_I^{(\alpha)}}\,, \tag{5.9}$$

where

$$f_{\rm w}(t,z) = A_{\rm w} t + H_{\rm w}(z) - 1\,, \quad h_{\rm m}(t,z) = A_{\rm m} t + H_{\rm m}(z)\,. \tag{5.10}$$

$H_{\rm w}$ and $H_{\rm m}$ are harmonic functions on Z space, and $\mathcal{A}_i^{(\rm m)}$ satisfies eqs. (C.13) and (C.15).

We give one concrete example, i.e. the M2-M5 brane system with M-wave (M2-M5-W).

If M5 brane is time-dependent (M2-M5-W (b): See table 5 for the configuration), the metric is then given by

$$\begin{aligned} ds^2 = & \, h_{\tilde{5}}^{2/3} H_2^{1/3} \Big[(h_{\tilde{5}} H_2)^{-1} \left\{ -dt^2 + (d\zeta^{\tilde{1}})^2 + f_{\rm w}(dt - d\zeta^{\tilde{1}})^2 \right\} \\ & + H_2^{-1} (dy^2)^2 + h_{\tilde{5}}^{-1} \sum_{\tilde{\alpha}=3}^{6} (dx^{\tilde{\alpha}})^2 + u_{ij} dz^i dz^j \Big]\,, \end{aligned} \tag{5.11}$$

that is

$$\mathcal{A} = h_{\tilde{5}}^{2/3} H_2^{1/3}\,, \quad g_0 = (h_{\tilde{5}} H_5)^{-1}\,, \quad g_{\tilde{2}} = g_{\tilde{3}} = g_{\tilde{4}} = g_{\tilde{5}} = h_{\tilde{5}}^{-1}\,, \quad g_2 = H_2^{-1}, \tag{5.12}$$

where

$$h_{\tilde{5}} = A_{\tilde{5}} t + H_{\tilde{5}}(z)\,. \tag{5.13}$$

The form field is given by

$$F_{(4)} = - *_{\rm Z} (dh_{\tilde{5}}) \wedge dy^2 + dH_2^{-1} \wedge dt \wedge d\zeta^{\tilde{1}} \wedge dy^2\,, \tag{5.14}$$

where $*_{\rm Z}$ is the Hodge dual operator in the four-dimensional Z space.

Since the classification of static solutions is given in [33, 34] and ours is basically the same, we discuss only interesting cases here. As we discussed in section 4, we can apply the present solutions to analyze cosmology and black holes. In order to discuss those subjects, we need either an isotropic and homogeneous three space in the brane world-volume or three-dimensional (or higher-dimensional) vacuum Z space. However, as seen in table 4, a wave breaks the isotropy and homogeneity in one wave-propagating dimension, and KK-monopoles not only give inhomogeneities but also fill branes in many dimensions after compactifying ζ-direction. These facts make the application to our interesting subjects more difficult as the number of KK-monopoles increases. As a result, we find several examples in the system of small number of branes, but few examples for the system of large number of branes. In appendix D, we present the brane configurations for those possible models

	branes	dim(Z)	β_c
case 1 ($\tilde{I}$ =M5)	M5-W(a)	5	$-1/2$
	M5-KKM(a)	3	0
	M5-KKM(c)-1	3	1/5
	M5-KKM(c)-2	3	0
	M2-M5-W(b)	4	$-1/5$
	M2-M5-KKM(b)	3	0
	M2-M5-KKM(e)-1	3	1/5
	M2-M5-KKM(e)-2	3	0
	M2-M5-W-KKM(b)	3	0
	M5-M5-KKM(a)	3	0
	M5-M5-KKM(b)	3	1/5
	M5-M5-KKM-KKM(a)	4	1/13
case 2 ($\tilde{I}$ =M2)	M2-KKM(a)	3	0
	M2-KKM(c)	3	1/3
	M2-M5-W(a)	4	$-1/5$
	M2-M5-KKM(a)	3	0
	M2-M5-KKM(d)-1	3	1/3
	M2-M5-KKM(d)-2	3	1/3
	M2-M5-W-KK(a)	3	0
	M2-M2-KKM(c)	3	1/3
	M2-M2-KKM(d)	3	0

Table 6. The power exponent β_c of the scale factor a of possible 4-dimensional cosmological model is given, i.e. $a \propto \tau_c^{\beta}$, where τ is the cosmic time. The labelling (a), (b), $\cdots$ corresponds to the configuration given in tables 4 and 16.

5.1 Cosmology

For zero M-brane or one M-brane systems, which we show in table 4, we can discuss cosmology for many cases. The possible cosmological models are marked by $\surd$ in the column "cos" (M5-W, M2-KKM, M5-KKM, W-KKM, KKM-KKM brane systems). For two M-brane system, the possible models are M2-M5-W, M2-M5-KKM, M2-M5-W-KKM, M2-M2-KKM, M5-M5-KKM, M5-M5-KKM-KKM. For more than two brane system with M-waves or KK-monopoles, we have no interesting case. Note that we have only three cases (M2-W, M5-W, and M2-M5-W) in which spacetime is regular on the branes (at $\boldsymbol{z} = \boldsymbol{z}_k$). For other configurations, the curvature diverges. One may need some mechanism to avoid singularity if our world is confined on the brane.

In table 6, we summarize the power exponent of the scale factor a of the expanding universe when the brane is time-dependent. For the case that the wave or KK-monopole is time-dependent, we always find the same power exponents, i.e,, $\beta_c = 1/3$ and $-1/3$ for the time-dependent wave and the time-dependent KK-monopole, respectively. In some cases [M2-KKM(c), M2-M5-KKM(d)-1,2, M2-M2-KKM(d), and the time-dependent wave (M5-W(b), W-KKM(a), M2-M5-W(c), M2-M5-W-KKM(c))], we find that the power exponent of the scale factor is 1/3, which is that of the expanding universe with stiff matter fluid.

We give a simple example of M2-M5-KKM(a). The metric is given by

$$ds^2 = h_{\tilde{2}}^{1/3} H_5^{2/3}\Big[(h_{\tilde{2}} H_5)^{-1}\left\{-dt^2 + (dy^1)^2\right\} + h_{\tilde{2}}^{-1}(dy^2)^2 + H_5^{-1}\sum_{\tilde{\alpha}=3}^{6}(dx^{\tilde{\alpha}})^2$$

	branes	d	$\beta_{\rm BH}$	BH
case 1 ($\tilde{I}$ =M5)	M5-W	6	1/5	
	M5-KKM	4	1/3	
	M2-M5-W	5	1/4	$\surd$
	M2-M5-KKM	4	1/3	
	M2-M5-W-KKM	4	1/3	$\surd$
case 2 ($\tilde{I}$ =M2)	M2-W	9	1/8	
	M2-KKM	4	1/3	
	M2-M5-W	5	1/4	$\surd$
	M2-M5-KKM	4	1/3	
	M2-M5-W-KKM	4	1/3	$\surd$
	M2-M2-KKM	4	1/3	
case 3 ($\tilde{I}$ =W)	M5-W	6	1/5	
	W-KKM	4	1/3	
	M2-M5-W	5	1/4	$\surd$
	M2-M5-W-KKM	4	1/3	$\surd$
case 4 ($\tilde{I}$ =KKM)	M5-KKM	4	1/3	
	W-KKM	4	1/3	
	M2-M5-KKM	4	1/3	
	M2-M5-W-KKM	4	1/3	$\surd$
	M2-M2-KKM	4	1/3	

Table 7. The power exponent $\beta_{\rm BH}$ of the scale factor $a_{\rm BH}$ of the asymptotic FRW universe for the possible 4-dimensional black hole spacetime is given, i.e. $a_{\rm BH} \propto \tau^{\beta_{\rm BH}}$, where τ is the cosmic time. The marked one in the column "BH" has a finite horizon area, i.e,, it has a regular horizon.

$$+h_m^{-1}(d\zeta + \mathcal{A}_i dz^i)^2 + h_m u_{ij} dz^i dz^j\Big]. \tag{5.15}$$

The compactified metric in the Einstein frame is

$$ds_4^2 = h_m^{-1/2}\left[-h_{\tilde{2}}^{-1}dt^2 + d\boldsymbol{\xi}^2\right]. \tag{5.16}$$

This gives $\beta_{\rm c} = 0$ in table 6.

For the case with the wave, we can smear some dimensions (<dim(Z)) just as in section 4.1, and the result is exactly the same as the case without the wave.

5.2 Time-dependent black holes

We can also discuss some black hole spacetime by compactifying the brane world-volume as in section 4.2. Although the spacetime is time dependent, near-brane geometry is the same as that of the static brane solution. If we find the horizon at $|\boldsymbol{z}| = 0$ for the static brane solution, we obtain a black hole geometry by compactification. As for the possible spacetime for a black hole (or object), we summarize our result in table 7.

We show one concrete example of M2-M5-W brane system. The metric is given by eq. (5.11). Compactifying the brane world-volume, the 5-dimensional metric in the Einstein frame is given by

$$d\bar{s}_5^2 = -\left[h_{\tilde{5}}H_2\left(1+f_{\rm w}\right)\right]^{-2/3}dt^2 + \left[h_{\tilde{5}}H_2\left(1+f_{\rm w}\right)\right]^{1/3}\left(dr^2 + r^2 d\Omega_3^2\right), \tag{5.17}$$

where

$$h_{\tilde{5}} = \frac{t}{t_0} + \frac{Q_{\tilde{5}}}{r^2}, \quad H_2 = 1 + \frac{Q_2}{r^2}, \quad f_{\rm w} = \frac{Q_{\rm w}}{r^2}, \tag{5.18}$$

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This metric is rewritten as

$$d\bar{s}_5^2 = -\left[\tilde{H}_{\tilde{5}} H_2 \left(1 + f_{\rm w}\right)\right]^{-2/3} d\tau^2 + a_{\rm BH}^2(\tau) \left[\tilde{H}_{\tilde{5}} H_2 \left(1 + f_{\rm w}\right)\right]^{1/3} \left(dr^2 + r^2 d\Omega_3^2\right) , \tag{5.19}$$

where

$$\tilde{H}_{\tilde{5}} = 1 + \frac{\tilde{Q}_{\tilde{5}}(\tau)}{r^2} , \quad \text{and} \quad a_{\rm BH} = \left(\frac{\tau}{\tau_0}\right)^{\frac{1}{4}} , \tag{5.20}$$

with

$$\tilde{Q}_{\tilde{5}} \equiv \left(\frac{\tau}{\tau_0}\right)^{-\frac{3}{2}} Q_{\tilde{5}} , \quad \text{and} \quad \tau_0 \equiv \frac{3}{2} t_0 . \tag{5.21}$$

The expansion rate of this scale factor is the same as that of the stiff-matter dominant universe in 5 dimensions. Hence this solution is regarded as a five-dimensional black hole in the expanding universe.

6 Lorentz invariance and the lower-dimensional effective theory

When we discuss four-dimensional cosmology, we assume that our three space is isotropic and homogeneous. However, in that case, the time direction can be different from three spatial directions. In fact if some branes are not filled in this three space, the time direction which is filled by all branes is not the same as three spatial directions. When we perform a Lorentz transformation, three spatial directions are not equivalent. For example, suppose we have M2-M5-M5 brane system. There are two possible cosmological models: the case 1 (M2-M5-M5 (b)) and the case 2 (M2-M5-M5 (a)) (see tables 2 and 10). We assume that we are living on three space $\boldsymbol{\xi} = (x^3, x^4, x^5)$. Then the four dimensional metric in the Einstein frame is

$$d\bar{s}_4^2 = F_{\tilde{I}}(z) \left[-f_0(z) dt^2 + f_\xi(z) d\boldsymbol{\xi}^2\right] , \tag{6.1}$$

where

$$\begin{array}{llll} F_{\tilde{2}} = (H_5 H_{5'})^{-1/2} \ , & f_0(z) = f_\xi(z) = (H_5 H_{5'})^{-1} & & \text{for M2-M5-M5 (a)} , \\ F_{\tilde{5}} = H_5^{-1/2} \ , & f_0(z) = (H_2 H_5)^{-1} \ , & f_\xi(z) = H_5^{-1} & \text{for M2-M5-M5 (b)} . \end{array} \tag{6.2}$$

For M2-M5-M5 (a), we have

$$d\bar{s}_4^2 = (H_5 H_{5'})^{-3/2} \left(-dt^2 + d\boldsymbol{\xi}^2\right) \propto \eta_{\mu\nu} d\xi^\mu d\xi^\nu , \tag{6.3}$$

where $\xi^\mu = (t, \boldsymbol{\xi})$. This spacetime is Lorentz invariant. On the other hand, for M2-M5-M5 (b), we find

$$d\bar{s}_4^2 = H_5^{-3/2} \left(-H_2^{-1} dt^2 + d\boldsymbol{\xi}^2\right) . \tag{6.4}$$

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When we perform a Lorentz transformation in the t-ξ plane ($\xi = \xi^1$);

$$\begin{pmatrix} t' \\ \xi' \end{pmatrix} = \begin{pmatrix} \gamma\,(t - V\xi) \\ \gamma\,(\xi - Vt) \end{pmatrix} , \tag{6.5}$$

where V is the velocity of new inertia frame and $\gamma = (1 - V^2)^{-1/2}$ is its Lorentz factor, we have

$$d\bar{s}_4^2 = H_5^{-3/2}\left[-(dt')^2 + (d\boldsymbol{\xi'})^2 + (H_2^{-1} - 1)\gamma^2(dt' + V d\xi')^2\right] . \tag{6.6}$$

The last term in eq. (6.6) gives the breaking term of Lorentz invariance. Hence the order of magnitude of breaking the Lorentz invariance is

$$\mathcal{O}\left(H_2^{-1} - 1\right) \sim \mathcal{O}\left(\sum_k \frac{Q_{2,k}}{|\boldsymbol{z} - \boldsymbol{z}_k|}\right) . \tag{6.7}$$

In order to keep the Lorentz invariance, we need the condition of $f_0 = f_\xi$, which means that all branes except for one time-dependent brane contain our three space Ξ. Hence 4D universes constructed from M2-M5 (a), M5-M5, M5-M5-M5, and M2-M5-M5 (a) have the Lorentz invariance.

We can extend this four-dimensional Minkowski space into a curved space with general covariance. We take the following metric just as in appendix A:

$$ds^2 = \mathcal{A}\left[g_0 q_{\mu\nu} d\xi^\mu d\xi^\nu + \sum_{\tilde{\alpha}\notin\Xi,\,\tilde{\alpha}\in\tilde{I}} g_{\tilde{I}}(dx^{\tilde{\alpha}})^2 + \sum_{\alpha\notin\tilde{I}} g_\alpha(dy^\alpha)^2 + u_{ij}dz^i dz^j\right] , \tag{6.8}$$

where we assume that the four-dimensional metric $q_{\mu\nu}$ depends only on ξ^μ and that all branes (or all except for one time-dependent brane) fill our four-dimensional spacetime. Inserting this metric form, we find the solution

$$\begin{aligned}
R_{\mu\nu}\left(\hat{\Xi}\right) &= 0 \ , \qquad\qquad R_{ij}(\mathrm{Z}) = 0, \\
h_{\tilde{I}} &= K_{\tilde{I}}(\xi) + H_{\tilde{I}}(z) \ , \quad h_I = H_I(z) \text{for} \ \ I \neq \tilde{I}, \\
D_\mu D_\nu K_{\tilde{I}} &= 0, \\
\triangle_{\mathrm{Z}} H_{\tilde{I}} &= 0 \ , \qquad\qquad \triangle_{\mathrm{Z}} H_I = 0 \ \ \text{for} I \neq \tilde{I} \,,
\end{aligned} \tag{6.9}$$

where $\hat{\Xi}$ is our four-dimensional spacetime.

Here let us point out the important fact on the nature of the dynamical solutions described above. We often discuss the four-dimensional effective theories, which are derived from eleven-dimensional supergravity with branes. In these occasions, we assume that there exists a Lorentz invariance in the limit of a flat Minkowski space, write down the scalar curvature, and then integrate the eleven-dimensional action over compactified extra dimensions to discuss the four-dimensional effective theories.

However, it has been well known for decades that such a procedure is generically inconsistent: one should work at the level of the field equations, write the field equations in lower-dimensional form and then integrate them back to a lower-dimensional action. In our specific case too, solutions of the effective four-dimensional theories may not give solutions of the above type with the warp factors in eleven dimensions which depend on time and z^i. This is because they are genuinely D-dimensional so that one can never neglect the dependence on the bulk space (Z) in the basic equations. Hence the solutions of the effective theories are quite often inconsistent with the basic equations in eleven dimensions. This has been recently shown explicitly for a single brane in ref. [9]. Here we would like to point out that this is also true in the intersecting brane solutions. This fact should be kept in mind because recently the dynamics of the internal space moduli in higher-dimensional theories has been investigated by using four-dimensional effective theories.

Let us show one explicit example of M5-M5 brane system. The four-dimensional effective action is obtained by dimensional reduction from our eleven-dimensional action. Our solution is

$$\begin{aligned} ds^2 &= (h_{\tilde{5}} H_5)^{-1/3} \Big[q_{\mu\nu}(\hat{\Xi}) d\xi^\mu d\xi^\nu + H_5 r_{\tilde{\alpha}\tilde{\beta}}(\mathrm{X}) dx^{\tilde{\alpha}} dx^{\tilde{\beta}} \\ &\qquad + h_{\tilde{5}} s_{\alpha\beta}(\mathrm{Y}) dy^\alpha dy^\beta + h_{\tilde{5}} H_5 u_{ab}(\mathrm{Z}) dz^a dz^b \Big] , \\ F_{(4)} &= - *_{\mathrm{Z}} (dh_{\tilde{5}}) \wedge dx^{\tilde{\alpha}} \wedge dx^{\tilde{\beta}} - *_{\mathrm{Z}} (dH_5) \wedge dy^\alpha \wedge dy^\beta , \end{aligned} \tag{6.10}$$

where $*_{\mathrm{Z}}$ is the Hodge dual operator in the Z space, and $q_{\mu\nu}(\hat{\Xi})$, $r_{\tilde{\alpha}\tilde{\beta}}(\mathrm{X})$, $s_{\alpha\beta}(\mathrm{Y})$ and $u_{ij}(\mathrm{Z})$ are the metrics on our four-dimensional spacetime $\hat{\Xi}$, two-dimensional space X, two-dimensional space Y and the three dimensional transverse space Z. Taking into account the ansatz $h_{\tilde{5}} = K_{\tilde{5}}(\xi) + H_{\tilde{5}}(z)$ and $H_5(z)$, we find the eleven-dimensional scalar curvature R is

$$\begin{aligned} R &= (h_{\tilde{5}} H_5)^{1/3} R(\hat{\Xi}) + (h_{\tilde{5}} H_5)^{-2/3} \left[h_{\tilde{5}} R(\mathrm{X}) + H_5 R(\mathrm{Y}) + R(\mathrm{Z}) \right] \\ &\quad - \frac{5}{3} (h_{\tilde{5}} H_5)^{1/3} h_{\tilde{5}}^{-1} \Box_{\hat{\Xi}} K_{\tilde{5}} - \frac{4}{3} (h_{\tilde{5}} H_5)^{-2/3} \left(h_{\tilde{5}}^{-1} \triangle_{\mathrm{Z}} h_{\tilde{5}} + H_5^{-1} \triangle_{\mathrm{Z}} H_5 \right) \\ &\quad + \frac{15}{18} (h_{\tilde{5}} H_5)^{-2/3} u^{ij} \left(\partial_i \ln h_{\tilde{5}} \partial_j \ln h_{\tilde{5}} + \partial_i \ln H_5 \partial_j \ln H_5 \right) , \end{aligned} \tag{6.11}$$

where $\Box_{\hat{\Xi}}$ and $\triangle_{\mathrm{Z}}$ are the D'Alembertian and Laplace operator for $\hat{\Xi}$-spacetime and Z-space, respectively. Assuming Ricci flatness for X, Y and Z spaces, and harmonicity for $H_{\tilde{5}}$ and H_5 ($\triangle_{\mathrm{Z}} H_{\tilde{5}} = \triangle_{\mathrm{Z}} H_5 = 0$), we get

$$S = \frac{1}{2\tilde{\kappa}^2} \int_{\hat{\Xi}} H(\xi) R(\hat{\Xi}) *_{\hat{\Xi}} \mathbf{1}_{\hat{\Xi}}, \tag{6.12}$$

where $*_{\hat{\Xi}}$ denotes the Hodge dual operator on $\hat{\Xi}$, we have dropped the surface terms, $\tilde{\kappa} \equiv (V_{\mathrm{X}} V_{\mathrm{Y}} V_0)^{-1/2} \kappa$, and $H(\xi)$ is defined by

$$K(\xi) = K_{\tilde{5}}(\xi) + \bar{c}; \quad \bar{c} := V_0^{-1} \int_{\mathrm{Z}} H_{\tilde{5}} H_5 *_{\mathrm{Z}} \mathbf{1}_{\mathrm{Z}}, \tag{6.13}$$

where $*_{\mathrm{Z}}$ represents the Hodge dual operator on Z, and V_{X}, V_{Y}, V_0 are given by

$$V_{\mathrm{X}} = \int_{\mathrm{X}} *_{\mathrm{X}} \mathbf{1}_{\mathrm{X}}, \quad V_{\mathrm{Y}} = \int_{\mathrm{Y}} *_{\mathrm{Y}} \mathbf{1}_{\mathrm{Y}}, \quad V_0 = \int_{\mathrm{Z}} H_5 *_{\mathrm{Z}} \mathbf{1}_{\mathrm{Z}}. \tag{6.14}$$

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The four-dimensional field equations are then given by

$$\begin{aligned} R_{\mu\nu}\left(\hat{\Xi}\right) &= K^{-1}D_\mu D_\nu K, \\ \Box_{\hat{\Xi}}K &= 0. \end{aligned} \tag{6.15}$$

If the four-dimensional spacetime $\hat{\Xi}$ is Ricci flat, these equations reproduce the correct ones for $K_{\tilde{5}}(x) = K - \bar{c}$ obtained from the eleven-dimensional theory before. However, the Ricci flatness of $\hat{\Xi}$ is not required in the present effective theory (6.12) unlike in the full eleven-dimensional theory (2.1). Hence, the class of solutions obtained in the four-dimensional effective theory are much larger than the higher-dimensional original theory [9]. This makes the higher-dimensional solutions even more restrictive than those of the four-dimensional effective equations. This is because the information of the internal space which gives constraints on the lower dimensions was lost after compactifying the internal space.

We note that the effective theory has a modular invariance similar to the no-flux case $F_4 = 0$. In fact, by the conformal transformation $ds^2_{\bar{\Xi}} = K^{-1}ds^2_{\hat{\Xi}}$, (6.12) is expressed in terms of the variables in the Einstein frame as

$$S = \frac{1}{2\tilde{\kappa}^2}\int_{\bar{\Xi}}\left[R(\bar{\Xi}) *_{\bar{\Xi}} \mathbf{1}_{\bar{\Xi}} - \frac{3}{2}d\varphi \wedge *_{\bar{\Xi}}d\varphi\right], \tag{6.16}$$

where $R(\bar{\Xi})$ is the scalar curvature with respect to the metric $ds^2_{\bar{\Xi}}$, $*_{\bar{\Xi}}$ denotes the Hodge dual operator on $\bar{\Xi}$, and $\varphi \equiv \sqrt{\frac{3}{2}}\ln K$. The corresponding four-dimensional Einstein equations in the Einstein frame and the field equation for φ are given by

$$\begin{aligned} R_{\mu\nu}(\bar{\Xi}) &= \bar{D}_\mu\varphi\,\bar{D}_\nu\varphi, \\ \Box_{\bar{\Xi}}\varphi &= 0\,. \end{aligned} \tag{6.17}$$

It is clear that this action and the equations of motion are invariant under the transformation $\varphi \to -\varphi + \lambda$, where λ is an arbitrary constant.

7 Concluding remarks

In this paper, we have derived general intersecting dynamical brane solutions, given the complete classification of the intersecting M-branes, and discussed the dynamics of the higher-dimensional supergravity models with applications to cosmology and black hole physics. The solutions we have found are the spacetime-dependent solutions. These solutions were obtained by replacing a time-independent warp factor $h_{\tilde{I}} = H_{\tilde{I}}(z)$ of a supersymmetric solution by a time-dependent function $h_{\tilde{I}} = A_{\tilde{I}}t + H_{\tilde{I}}(z)$ [14, 27]. Our solutions can contain only one function depending on both time t and transverse space coordinates z^i.

Supposing that our universe stays at a constant position in the bulk space Z ($\boldsymbol{z}_k$), we have shown that several four-dimensional effective theories on the branes give four-dimensional Minkowski space or FRW universe. The power of the scale factor, however, is too small to give a realistic expansion law. This means that we have to consider additional matter on the brane in order to get a realistic expanding universe. On the other hand, we can also discuss time-dependent black hole spacetimes which approach asymptotically the

FRW universe, if we regard the bulk space as our universe. The near horizon geometries of these black holes in the expanding universe are the same as the static solutions. However the asymptotic structures are completely different, giving the FRW universe with scale factors same as the universe filled by stiff matter.

In the viewpoint of higher-dimensional theory, the dynamics of four-dimensional background are given by the solution of higher-dimensional Einstein equations. For instance, in the black p-brane system, the solution tells us that the $(p+1)$-dimensional spacetime X is Ricci flat. The $(p+1)$-dimensional spacetime is then similar to the Kasner solution for the $(p+1)$-dimensional background [14]. On the other hand, if we start from the lower-dimensional effective theory for warped compactification, the solutions may not be allowed in the higher-dimensional theory. We have shown that it is the case in M5-M5 brane system. The same is true for M5-M5-M5 and D2-D6 brane systems in ten-dimensional type IIA theories. This is because the function of z in the metric is integrated out in the lower-dimensional effective action. Then, the information of the extra dimensions in the function h of the metric will be lost by the compactification. This result implies that we have to be careful when we use a four-dimensional effective theory to analyse the moduli stabilization problem and the cosmological problems in the framework of warped compactification of supergravity or M-theory [9] (see also [10, 11, 40–43] for recent progress in the effective theory for warped compactifications).

We have also noted that if the Lorentz invariance is not kept on the lower-dimensional world-sheet, the lower-dimensional effective action cannot be written in the covariant form for the lower-dimensional metric. Some of the four- or five-dimensional effective theories in this paper thus have broken Lorentz invariance on the world-sheet. Although the examples considered in the present paper do not provide realistic cosmological models, this feature may be utilised to investigate a cosmological analysis in a realistic higher-dimensional cosmological model.

As we stated above, our solutions can contain only one function depending on both time and transverse space coordinates, and this seems to be a limitation on the applications of the solutions. Recent study of similar systems depending on the light-cone coordinate and space shows that it is possible to obtain solutions with more nontrivial dependence on spacetime coordinates [44]. It is interesting to study if similar more general solutions can be obtained by relaxing some of our ansätze. We hope to report on this subject in the near future.

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A Dynamical solution of a single p-brane

In this appendix, we briefly summarize the results for the case of a single dynamical p-brane [14]. We consider a single p-brane in our action (2.1) [2]. In what follows, we use the same notation for the variables and parameters of this single brane dropping the suffix I.

To solve the field equations (2.3a), (2.3b), and (2.3c), we assume the D-dimensional metric in the form

$$ds^2 = h^a(x,z)u_{ij}(\mathrm{Z})dz^i dz^j + h^b(x,z)q_{\mu\nu}(\mathrm{X})dx^\mu dx^\nu, \tag{A.1}$$

where $q_{\mu\nu}$ is a $(p+1)$-dimensional metric which depends only on the coordinates $x^\mu \equiv (t, x^\alpha)$ with α being the spatial coordinates of the brane, and u_{ij} is the $(D-p-1)$-dimensional metric which depends only on the coordinates z^i. The parameters a and b are given in eqs. (2.6). Note that in the case of interacting branes, we divide the coordinate for branes into two parts; the time coordinate t and the spatial coordinates of brane world-volume x^α, and assume that the metric depends on only t and z^i, but not on x^α. The metric form (A.1) is a straightforward generalization of the case of a static p-brane system with a dilaton coupling [2].

We also assume that the scalar field ϕ and the gauge field strength $F_{(p+2)}$ are given by eq. (2.7)

With the above ansatz, the Einstein equations are given by

$$R_{\mu\nu}(\mathrm{X}) - h^{-1}D_\mu D_\nu h - \frac{b}{2}h^{-1}q_{\mu\nu}\left(\triangle_{\mathrm{X}}h + h^{-1}\triangle_{\mathrm{Z}}h\right) = 0, \tag{A.2a}$$

$$R_{ij}(\mathrm{Z}) - \frac{a}{2}u_{ij}\left(\triangle_{\mathrm{X}}h + h^{-1}\triangle_{\mathrm{Z}}h\right) = 0, \tag{A.2b}$$

$$\partial_\mu\partial_i h = 0, \tag{A.2c}$$

where D_μ is the covariant derivative with respective to the metric $q_{\mu\nu}$, $\triangle_{\mathrm{X}}$ and $\triangle_{\mathrm{Z}}$ are the Laplace operators on the space of X and the space Z, and $R_{\mu\nu}(\mathrm{X})$ and $R_{ij}(\mathrm{Z})$ are the Ricci tensors of the metrics $q_{\mu\nu}$ and u_{ij}, respectively. From eq. (A.2c), the warp factor h must be in the form

$$h(x,z) = K(x) + H(z). \tag{A.3}$$

With this form of h, the other components of the Einstein equations (A.2a) and (A.2b) are rewritten as

$$R_{\mu\nu}(\mathrm{X}) - h^{-1}D_\mu D_\nu K - \frac{b}{2}h^{-1}q_{\mu\nu}\left(\triangle_{\mathrm{X}}K + h^{-1}\triangle_{\mathrm{Z}}H\right) = 0, \tag{A.4a}$$

$$R_{ij}(\mathrm{Z}) - \frac{a}{2}u_{ij}\left(\triangle_{\mathrm{X}}K + h^{-1}\triangle_{\mathrm{Z}}H\right) = 0. \tag{A.4b}$$

Next we consider the gauge field. Under the assumption (2.7), we find

$$dF_{(p+2)} = h^{-1}(2\partial_i \ln h \partial_j \ln h + h^{-1}\partial_i\partial_j h)dz^i \wedge dz^j \wedge \Omega(\mathrm{X}) = 0. \tag{A.5}$$

Thus, the Bianchi identity is automatically satisfied. Also the equation of motion for the gauge field (2.3c) becomes

$$d\left[e^{-c\phi} * F_{(p+2)}\right] = -d\left[\partial_i h(*_{\mathrm{Z}}dz^i)\right] = 0, \tag{A.6}$$

where $*_{\rm Z}$ denotes the Hodge dual operator on Z. Hence, the gauge field equation is automatically satisfied.

Let us consider the scalar field equation. Substituting the forms of the scalar field and the gauge field (eq. (2.7)), and the warp factor (A.3) into the equation of motion for the scalar field (2.3b), we obtain

$$\frac{c}{2}h^{-a}\left(\triangle_{\rm X}K + h^{-1}\triangle_{\rm Z}H\right) = 0, \tag{A.7}$$

Thus, unless the parameter c is zero, the warp factor h should satisfy the equations

$$\triangle_{\rm X}K = 0, \quad \triangle_{\rm Z}H = 0. \tag{A.8}$$

If $F_{(p+2)} \neq 0$, the function H is non-trivial. In this case, the Einstein equations reduce to

$$\begin{aligned} R_{\mu\nu}({\rm X}) &= 0, \\ R_{ij}({\rm Z}) &= 0, \\ D_\mu D_\nu K &= 0. \end{aligned} \tag{A.9}$$

If $F_{(p+2)} = 0$, however, the function H becomes trivial, and then the internal space is no longer warped [9].

We show an example. We consider the case

$$q_{\mu\nu} = \eta_{\mu\nu}\,, \quad u_{ij} = \delta_{ij}\,, \tag{A.10}$$

that is, we have the $(p+1)$-dimensional Minkowski space and the $(D-p-1)$-dimensional Euclidean space. In this case, the solution for h is obtained explicitly as

$$h(x,z) = A_\mu x^\mu + B + \sum_k \frac{Q_k}{|\boldsymbol{z}-\boldsymbol{z}_k|^{D-p-3}}, \tag{A.11}$$

where A_μ, B and Q_k are constant parameters.

For the case of $c = 0$, the scalar field becomes constant because of the ansatz (2.7), and the scalar field equation (A.7) is automatically satisfied. Then, the Einstein equations become

$$\begin{aligned} R_{\mu\nu}({\rm X}) &= 0, \\ R_{ij}({\rm Z}) &= \frac{1}{2}a(p+1)\,\lambda\, u_{ij}({\rm Z}), \\ D_\mu D_\nu K &= \lambda\, q_{\mu\nu}({\rm X}), \end{aligned} \tag{A.12}$$

where λ is a constant. We see that the internal space Z is not Ricci flat, but the Einstein space if $\lambda \neq 0$, and the function K can be more non-trivial. For example, if $q_{\mu\nu} = \eta_{\mu\nu}$, K is no longer linear but quadratic in the coordinates x^μ [27].

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		0	1	2	3	4	5	6	7	8	9	10	$\tilde{I}$	cos	BH
$(\mathrm{M2})^2$	M2	$\circ$	$\circ$	$\circ$									(a)	–	$\surd$
	M2	$\circ$			$\circ$	$\circ$									
M2M5	M2	$\circ$	$\circ$					$\circ$					(a)	$\surd$	$\surd$
	M5	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$						(b)	$\surd$	$\surd$
$(\mathrm{M5})^2$	M5	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$						(a)	$\surd$	$\surd$
	M5	$\circ$	$\circ$	$\circ$	$\circ$			$\circ$	$\circ$						

Table 8. Intersections of two M-branes.

B Classification of intersecting branes

In this appendix, we present a complete classification of time-dependent intersecting M-branes. For two $\sim$ four brane systems, we give all possible brane configurations and those metrics explicitly in tables 8–15. In the first tables in these tables, circles indicate where the brane world-volumes enter, and the time-dependent branes are indicated by (a) and (b) for different solutions. When the solutions can be used for cosmology and black hole physics, they are marked in the corresponding columns. In the second (continued) tables, concrete metrics are given in the notation of section 3 with the dimensions of transverse space Z for each time-dependent case indicated in the first tables by (a) and (b).

For more than four branes, we show only simplified tables (tables 16–19) to save the space because these systems do not have applications to cosmology and black hole physics, and are not so interesting. They are included for the sake of completeness. In these tables, we show which branes are involved, and dimension of the transverse space Z, and the following columns with kM give the numbers of dimensions containing k branes. For example, (2, 2, 2, 2, 1) in the first row of table 16 means that there are these numbers of dimensions in which the world-volumes of 1 M-brane, 2 M-branes and so on lie. Though these are not so explicit, they are useful to identify the explicit brane configurations with higher numbers of branes from the systems with lower numbers step by step. In the next column is given how many different time-dependent solutions are obtained according to which brane we give the time dependence. For example, M5(3) in the first column of table 16 means that there are only three kinds of different solutions when we choose different time-dependent M5 branes. This is because there are same kind of M5 branes which give the same time-dependent solutions. Which brane gives different time-dependent solutions can be easily identified if we check the patterns of how many branes each coordinate of the brane contains.

C Dynamical solution of KK-wave and KK-monopole

C.1 KK-wave

Here, we discuss the dynamical solution of of KK-wave. We start from $(D-1)$-dimensional spacetime, and consider the KK 2-form $\mathcal{F}_{AB}$ with a coupling to the dilaton. Replacing D

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M2M2		$\mathcal{A}=h_{\tilde{2}}^{1/3}H_2^{1/3}$	$g_{\tilde{0}}=h_{\tilde{2}}^{-1}H_2^{-1}$
	dim(Z)	$g_{\tilde{\alpha}}$	g_α
(a)	5	$g_{\tilde{1}}=g_{\tilde{2}}=h_{\tilde{2}}^{-1}$	$g_3=g_4=H_2^{-1}$
M2M5		$\mathcal{A}=h_{\tilde{2}}^{1/3}H_5^{2/3}$	$g_0=h_{\tilde{2}}^{-1}H_5^{-1}$
	dim(Z)	$g_{\tilde{\alpha}}$	g_α
(a)	4	$g_{\tilde{1}}=h_{\tilde{2}}^{-1}H_5^{-1}, g_{\tilde{6}}=h_{\tilde{2}}^{-1}$	$g_2=g_3=g_4=g_5=H_5^{-1}$
M2M5		$\mathcal{A}=h_{\tilde{5}}^{2/3}H_5^{1/3}$	$g_0=h_{\tilde{5}}^{-1}H_5^{-1}$
	dim(Z)	$g_{\tilde{\alpha}}$	g_α
(b)	4	$g_{\tilde{1}}=h_{\tilde{5}}^{-1}H_2^{-1}$ $g_{\tilde{2}}=g_{\tilde{3}}=g_{\tilde{4}}=g_{\tilde{5}}=h_{\tilde{5}}^{-1}$	$g_6=H_2^{-1}$
M5M5		$\mathcal{A}=h_{\tilde{2}}^{2/3}H_2^{2/3}$	$g_{\tilde{0}}=h_{\tilde{2}}^{-1}H_2^{-1}$
	dim(Z)	$g_{\tilde{\alpha}}$	g_α
(a)	3	$g_{\tilde{1}}=g_{\tilde{2}}=g_{\tilde{3}}=h_{\tilde{5}}^{-1}H_2^{-1}$ $g_{\tilde{4}}=g_{\tilde{5}}=h_{\tilde{2}}^{-1}$	$g_6=g_7=H_5^{-1}$

Table 9. (Continue) Concrete metrics for two M-branes.

		0	1	2	3	4	5	6	7	8	9	10	$\tilde{I}$	cos	BH
$(\mathrm{M5})^3$	M5	◦	◦	◦	◦	◦	◦						(a)	$\surd$	−
	M5	◦	◦	◦	◦			◦	◦						
	M5	◦	◦	◦	◦					◦	◦				
	M5	◦	◦	◦	◦	◦	◦						(b)	−	$\surd$
	M5	◦	◦	◦	◦			◦	◦						
	M5	◦	◦			◦	◦	◦	◦						
	M5	◦	◦	◦	◦	◦	◦						(c)	−	−
	M5	◦	◦	◦	◦			◦	◦						
	M5	◦	◦	◦		◦		◦		◦					
$\mathrm{M2(M5)}^2$	M2	◦	◦	◦									(a)	$\surd$	$\surd$
	M5	◦	◦		◦	◦	◦	◦					(b)	$\surd$	$\surd$
	M5	◦		◦	◦	◦	◦		◦						
	M2	◦	◦	◦									(a)	−	−
	M5	◦	◦		◦	◦	◦	◦					(b)	−	−
	M5	◦	◦				◦	◦	◦	◦					
$\mathrm{(M2)}^2\mathrm{M5}$	M2	◦	◦	◦									(a)	$\surd$	$\surd$
	M2	◦			◦	◦									
	M5	◦	◦		◦		◦	◦	◦				(b)	$\surd$	$\surd$
$\mathrm{(M2)}^3$	M2	◦	◦	◦									(a)	−	$\surd$
	M2	◦			◦	◦									
	M2	◦					◦	◦							

Table 10. Intersections of three M-branes.

with $(D-1)$ and substituting $p=0$ into eqs. (2.3a), (2.3b), (2.3c), (2.6), and (A.1), we find the electric 0-brane solution in $(D-1)$ dimensions written by

$$ds^2_{D-1} = -h_{\mathrm{w}}^{-\frac{D-4}{D-3}}dt^2 + h_{\mathrm{w}}^{\frac{1}{D-3}}\, u_{ij}(\mathrm{Z})\, dz^i dz^j, \tag{C.1a}$$

$$e^\phi = h_{\mathrm{w}}^{\sqrt{\frac{D-2}{2(D-3)}}}, \qquad \mathcal{A}^{(\mathrm{w})} = (h_{\mathrm{w}}^{-1}-1)\, dt, \tag{C.1b}$$

$$R_{ij}(\mathrm{Z}) = 0, \tag{C.1c}$$

$$h_{\mathrm{w}}(t,z) = K_{\mathrm{w}}(t) + H_{\mathrm{w}}(z), \quad K_{\mathrm{w}}(t) = A_{\mathrm{w}}t + B_{\mathrm{w}}, \quad \triangle_{\mathrm{Z}} H_{\mathrm{w}} = 0, \tag{C.1d}$$

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M5^3		$\mathcal{A} = h_{\tilde{5}}^{2/3} H_5^{2/3} H_{5'}^{2/3}$	$g_{\tilde{0}} = h_{\tilde{5}}^{-1} H_5^{-1} H_{5'}^{-1}$
	dim(Z)	$g_{\tilde{\alpha}}$	g_α
(a)	1	$g_{\tilde{1}} = g_{\tilde{2}} = g_{\tilde{3}} = h_{\tilde{5}}^{-1} H_5^{-1} H_{5'}^{-1}$ $g_{\tilde{4}} = g_{\tilde{5}} = h_{\tilde{5}}^{-1}$	$g_6 = g_7 = H_5^{-1}$ $g_8 = g_9 = H_{5'}^{-1}$
(b)	3	$g_{\tilde{1}} = h_{\tilde{5}}^{-1} H_5^{-1} H_{5'}^{-1}$ $g_{\tilde{2}} = g_{\tilde{3}} = h_{\tilde{5}}^{-1} H_5^{-1}$ $g_{\tilde{4}} = g_{\tilde{5}} = h_{\tilde{5}}^{-1} H_{5'}^{-1}$	$g_6 = g_7 = H_5^{-1} H_{5'}^{-1}$
(c)	2	$g_{\tilde{1}} = g_{\tilde{2}} = h_{\tilde{5}}^{-1} H_5^{-1} H_{5'}^{-1}$ $g_{\tilde{3}} = h_{\tilde{5}}^{-1} H_5^{-1}, g_{\tilde{4}} = h_{\tilde{5}}^{-1} H_{5'}^{-1}$ $g_{\tilde{5}} = h_{\tilde{5}}^{-1}$	$g_6 = H_5^{-1} H_{5'}^{-1}$ $g_7 = H_5^{-1}$ $g_8 = H_{5'}^{-1}$
M2M5^2		$\mathcal{A} = h_{\tilde{2}}^{1/3} H_5^{2/3} H_{5'}^{2/3}$	$g_0 = h_{\tilde{2}}^{-1} H_5^{-1} H_{5'}^{-1}$
	dim(Z)	$g_{\tilde{\alpha}}$	g_α
(a)	3	$g_{\tilde{1}} = h_{\tilde{2}}^{-1} H_5^{-1}$ $g_{\tilde{2}} = h_{\tilde{2}}^{-1} H_{5'}^{-1}$	$g_3 = g_4 = g_5 = H_5^{-1} H_{5'}^{-1}$ $g_6 = H_5^{-1}, g_7 = H_{5'}^{-1}$
(b)	2	$g_{\tilde{1}} = h_{\tilde{2}}^{-1} H_5^{-1} H_{5'}^{-1}$ $g_{\tilde{2}} = h_{\tilde{2}}^{-1}$	$g_3 = g_4 = H_5^{-1}$ $g_5 = g_6 = H_5^{-1} H_{5'}^{-1}$ $g_7 = g_8 = H_{5'}^{-1}$
M2M5^2		$\mathcal{A} = h_{\tilde{5}}^{2/3} H_2^{1/3} H_5^{2/3}$	$g_0 = h_{\tilde{5}}^{-1} H_2^{-1} H_5^{-1}$
	dim(Z)	$g_{\tilde{\alpha}}$	g_α
(c)	3	$g_{\tilde{1}} = h_{\tilde{5}}^{-1} H_2^{-1}$ $g_{\tilde{3}} = g_{\tilde{4}} = g_{\tilde{5}} = h_{\tilde{5}}^{-1} H_5^{-1}$ $g_{\tilde{6}} = h_{\tilde{5}}^{-1}$	$g_2 = H_2^{-1} H_5^{-1}$ $g_7 = H_5^{-1}$
(d)	2	$g_{\tilde{1}} = h_{\tilde{5}}^{-1} H_2^{-1} H_5^{-1}$ $g_{\tilde{3}} = g_{\tilde{4}} = h_{\tilde{5}}^{-1}$ $g_{\tilde{5}} = g_{\tilde{6}} = h_{\tilde{5}}^{-1} H_5^{-1}$	$g_2 = H_2^{-1}$ $g_7 = g_8 = H_5^{-1}$
M2^{2}M5		$\mathcal{A} = h_{\tilde{2}}^{1/3} H_2^{1/3} H_5^{2/3}$	$g_0 = h_{\tilde{2}}^{-1} H_2^{-1} H_5^{-1}$
	dim(Z)	$g_{\tilde{\alpha}}$	g_α
(a)	3	$g_{\tilde{1}} = h_{\tilde{2}}^{-1} H_5^{-1}$ $g_{\tilde{2}} = h_{\tilde{2}}^{-1}$ $g_{\tilde{1}} = h_{\tilde{5}}^{-1} H_2^{-1}$	$g_3 = H_2^{-1} H_5^{-1}$ $g_4 = H_2^{-1}$
M2^{2}M5		$\mathcal{A} = h_{\tilde{5}}^{2/3} H_2^{1/3} H_{2'}^{1/3}$	$g_0 = h_{\tilde{5}}^{-1} H_2^{-1} H_{2'}^{-1}$
	dim(Z)	$g_{\tilde{\alpha}}$	g_α
(b)	3	$g_{\tilde{1}} = h_{\tilde{5}}^{-1} H_2^{-1}$ $g_{\tilde{3}} = h_{\tilde{5}}^{-1} H_{2'}^{-1}$ $g_{\tilde{5}} = g_{\tilde{6}} = g_{\tilde{7}} = h_{\tilde{5}}^{-1}$	$g_2 = H_2^{-1}, g_4 = H_{2'}^{-1}$
M2^3		$\mathcal{A} = h_{\tilde{2}}^{1/3} H_2^{1/3} H_{2'}^{1/3}$	$g_0 = h_{\tilde{2}}^{-1} H_2^{-1} H_{2'}^{-1}$
	dim(Z)	$g_{\tilde{\alpha}}$	g_α
(a)	4	$g_{\tilde{1}} = g_{\tilde{2}} = h_{\tilde{2}}^{-1}$	$g_3 = g_4 = H_2^{-1}$ $g_5 = g_6 = H_{2'}^{-1}$

Table 11. (Continue) Concrete metrics for three M-branes.

where $\mathcal{F} = d\mathcal{A}^{(\mathrm{w})}$, and $R_{ij}(\mathrm{Z})$ and $\triangle_{\mathrm{Z}}$ are the Ricci tensor, and the Laplace operator with respect to the $(D-2)$-dimensional metric u_{ij}, and A_{w} and B_{w} are integration constants. Before going to D dimensions, we have to rescale the metric (C.1d) to put it in the D-dimensional Einstein frame. This is given by the conformal transformation

$$\bar{g}_{MN} = h_{\mathrm{w}}^{-1/(D-3)} g_{MN}. \tag{C.2}$$

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		0	1	2	3	4	5	6	7	8	9	10	$\tilde{I}$	cos	BH
$(\mathrm{M5})^4$	M5	∘	∘	∘	∘	∘	∘						(a)	–	–
	M5	∘	∘	∘	∘			∘	∘				(b)	–	–
	M5	∘	∘	∘		∘		∘		∘					
	M5	∘	∘		∘	∘			∘	∘					
	M5	∘	∘	∘	∘	∘		∘					(c)	–	–
	M5	∘	∘	∘	∘		∘		∘						
	M5	∘	∘	∘		∘	∘			∘					
	M5	∘	∘		∘	∘	∘				∘				
	M5	∘	∘	∘	∘	∘	∘						(d)	–	–
	M5	∘	∘	∘	∘			∘	∘				(e)	–	–
	M5	∘	∘	∘		∘		∘		∘					
	M5	∘	∘	∘			∘	∘			∘				
	M5	∘	∘	∘	∘	∘	∘						(f)	–	–
	M5	∘	∘	∘	∘			∘	∘						
	M5	∘	∘	∘		∘		∘		∘					
	M5	∘	∘	∘			∘		∘	∘					
M2 $(\mathrm{M5})^3$	M2	∘	∘	∘									(a)	–	–
	M5	∘	∘		∘	∘	∘	∘					(b)	–	–
	M5	∘	∘		∘	∘			∘	∘					
	M5	∘	∘				∘	∘	∘	∘					
	M2	∘	∘	∘									(c)	–	–
	M5	∘	∘		∘	∘		∘	∘				(d)	–	–
	M5	∘	∘		∘	∘	∘			∘					
	M5	∘	∘		∘		∘	∘			∘				
	M2	∘	∘	∘									(e)	–	–
	M5	∘	∘		∘	∘	∘		∘				(f)	–	–
	M5	∘	∘		∘	∘		∘		∘					
	M5	∘		∘	∘	∘	∘	∘					(g)	–	–

Table 12. Intersections of four M-branes I.

Gathering the above results, we find the D-dimensional metric of KK-wave [45]:

$$\begin{aligned} ds^2 = g_{MN}dx^M dx^N &= -h_{\mathrm{w}}^{-1}dt^2 + h_{\mathrm{w}}\left[d\zeta + (h_{\mathrm{w}}^{-1} - 1)dt\right]^2 + u_{ij}dz^i dz^j \\ &= -dt^2 + d\zeta^2 + f_{\mathrm{w}}\,(dt - d\zeta)^2 + u_{ij}dz^i dz^j\,, \end{aligned} \tag{C.3}$$

where u_{ij} denotes the $(D-2)$-dimensional metric depending only on the transverse coordinate z^i, while the function $h_{\mathrm{w}} \equiv 1 + f_{\mathrm{w}}$ can depend on both t and z^i.

Now we discuss the dynamical solution of Einstein equations for D-dimensional metric (C.3). Substituting the metric (C.3) into then vacuum Einstein equations, we obtain

$$-\triangle_{\mathrm{Z}}h_{\mathrm{w}} + (h_{\mathrm{w}} - 2)\partial_t^2 h_{\mathrm{w}} = 0, \tag{C.4a}$$

$$(1 - h_{\mathrm{w}})\partial_t^2 h_{\mathrm{w}} + \triangle_{\mathrm{Z}}h_{\mathrm{w}} = 0, \tag{C.4b}$$

$$\partial_t\partial_i h_{\mathrm{w}} = 0, \tag{C.4c}$$

$$h_{\mathrm{w}}\partial_t^2 h_{\mathrm{w}} - \triangle_{\mathrm{Z}}h_{\mathrm{w}} = 0, \tag{C.4d}$$

$$R_{ij}(\mathrm{Z}) = 0, \tag{C.4e}$$

where $\triangle_{\mathrm{Z}}$ and $R_{ij}(\mathrm{Z})$ are the Laplace operator and the Ricci tensor with respect to the metric u_{ij}, respectively. From (C.4c), we get

$$h_{\mathrm{w}}(t,z) = K_{\mathrm{w}}(t) + H_{\mathrm{w}}(z). \tag{C.5}$$

M5^4		$\mathcal{A}=(h_{\tilde{5}}H_5H_{5'}H_{5''})^{2/3}$	$g_{\tilde{0}}=(h_{\tilde{5}}H_5H_{5'}H_{5''})^{-1}$
	dim(Z)	$g_{\tilde{\alpha}}$	g_α
(a)	2	$g_{\tilde{1}}=(h_{\tilde{5}}H_5H_{5'}H_{5''})^{-1}, g_{\tilde{2}}=(h_{\tilde{5}}H_5H_{5'})^{-1}$ $g_{\tilde{3}}=(h_{\tilde{5}}H_5H_{5''})^{-1}, g_{\tilde{4}}=(h_{\tilde{5}}H_{5'}H_{5''})^{-1}$ $g_{\tilde{5}}=h_{\tilde{5}}^{-1}$	$g_6=(H_5H_{5'})^{-1}$ $g_7=(H_5H_{5''})^{-1}$ $g_8=(H_{5'}H_{5''})^{-1}$
(b)	2	$g_{\tilde{1}}=(h_{\tilde{5}}H_5H_{5'}H_{5''})^{-1}$ $g_{\tilde{2}}=(h_{\tilde{5}}H_5H_{5'})^{-1}, g_{\tilde{3}}=(h_{\tilde{5}}H_5H_{5''})^{-1},$ $g_{\tilde{6}}=(h_{\tilde{5}}H_{5'})^{-1}, g_{\tilde{7}}=(h_{\tilde{5}}H_{5''})^{-1}$	$g_4=(H_5H_{5'}H_{5''})^{-1}$ $g_5=H_5^{-1}$ $g_8=(H_{5'}H_{5''})^{-1}$
(c)	1	$g_{\tilde{1}}=(h_{\tilde{5}}H_5H_{5'}H_{5''})^{-1}, g_{\tilde{2}}=(h_{\tilde{5}}H_5H_{5'})^{-1}$ $g_{\tilde{3}}=(h_{\tilde{5}}H_5H_{5''})^{-1}, g_{\tilde{4}}=(h_{\tilde{5}}H_{5'}H_{5''})^{-1}$ $g_{\tilde{6}}=h_{\tilde{5}}^{-1}$	$g_5=(H_5H_{5'}H_{5''})^{-1}$ $g_7=H_5^{-1}, g_8=H_{5'}^{-1}$ $g_9=H_{5''}^{-1}$
(d)	1	$g_{\tilde{1}}=g_{\tilde{2}}=(h_{\tilde{5}}H_5H_{5'}H_{5''})^{-1}$ $g_{\tilde{3}}=(h_{\tilde{5}}H_5)^{-1}, g_{\tilde{4}}=(h_{\tilde{5}}H_{5'})^{-1}$ $g_{\tilde{5}}=(h_{\tilde{5}}H_{5''})^{-1}$	$g_6=(H_5H_{5'}H_{5''})^{-1}$ $g_7=H_5^{-1}, g_8=H_{5'}^{-1}$ $g_9=H_{5''}^{-1}$
(e)	1	$g_{\tilde{1}}=g_{\tilde{2}}=(h_{\tilde{5}}H_5H_{5'}H_{5''})^{-1}$ $g_{\tilde{3}}=(h_{\tilde{5}}H_5)^{-1}, g_{\tilde{6}}=(h_{\tilde{5}}H_{5'}H_{5''})^{-1}$ $g_{\tilde{7}}=(h_{\tilde{5}})^{-1}$	$g_4=(H_5H_{5'})^{-1}$ $g_5=(H_5H_{5''})^{-1}$ $g_8=H_{5'}^{-1}, g_9=H_{5''}^{-1}$
(f)	2	$g_{\tilde{1}}=g_{\tilde{2}}=(h_{\tilde{5}}H_5H_{5'}H_{5''})^{-1}$ $g_{\tilde{3}}=(h_{\tilde{5}}H_5)^{-1}, g_{\tilde{4}}=(h_{\tilde{5}}H_{5'})^{-1}$ $g_{\tilde{5}}=(h_{\tilde{5}}H_{5''})^{-1}$	$g_6=(H_5H_{5'})^{-1}$ $g_7=(H_5H_{5''})^{-1}$ $g_8=(H_{5'}H_{5''})^{-1}$
M2M5^3		$\mathcal{A}=h_{\tilde{2}}^{1/3}(H_5H_{5'}H_{5''})^{2/3}$	$g_0=(h_{\tilde{2}}H_5H_{5'}H_{5''})^{-1}$
	dim(Z)	$g_{\tilde{\alpha}}$	g_α
(a)	2	$g_{\tilde{1}}=(h_{\tilde{2}}H_5H_{5'}H_{5''})^{-1}$ $g_{\tilde{2}}=h_{\tilde{2}}^{-1}$	$g_3=g_4=H_5^{-1}H_{5'}^{-1}$ $g_5=g_6=H_5^{-1}H_{5''}^{-1}$ $g_7=g_8=H_{5'}^{-1}H_{5''}^{-1}$
(c)	1	$g_{\tilde{1}}=(h_{\tilde{2}}H_5H_{5'}H_{5''})^{-1}$ $g_{\tilde{2}}=h_{\tilde{2}}^{-1}$	$g_3=(H_5H_{5'}H_{5''})^{-1}, g_4=H_5^{-1}H_{5'}^{-1}$ $g_5=H_{5'}^{-1}H_{5''}^{-1}, g_6=H_5^{-1}H_{5''}^{-1}$ $g_7=H_5^{-1}, g_8=H_{5'}^{-1}, g_9=H_{5''}^{-1}$
(e)	2	$g_{\tilde{1}}=h_{\tilde{2}}^{-1}H_5^{-1}H_{5'}^{-1}$ $g_{\tilde{2}}=h_{\tilde{2}}^{-1}H_{5''}^{-1}$	$g_3=g_4=(H_5H_{5'}H_{5''})^{-1}$ $g_5=H_5^{-1}H_{5''}^{-1}, g_6=H_{5'}^{-1}H_{5''}^{-1}$ $g_7=H_5^{-1}, g_8=H_{5'}^{-1}$
M2M5^3		$\mathcal{A}=h_{\tilde{5}}^{2/3}H_2^{1/3}H_5^{2/3}H_{5'}^{2/3}$	$g_0=(h_{\tilde{5}}H_2H_5H_{5'})^{-1}$
	dim(Z)	$g_{\tilde{\alpha}}$	g_α
(b)	2	$g_{\tilde{1}}=(h_{\tilde{5}}H_2H_5H_{5'})^{-1}$ $g_{\tilde{3}}=g_{\tilde{4}}=h_{\tilde{5}}^{-1}H_5^{-1}$ $g_{\tilde{5}}=g_{\tilde{6}}=h_{\tilde{5}}^{-1}H_{5'}^{-1}$	$g_2=H_2^{-1}$ $g_7=g_8=H_5^{-1}H_{5'}^{-1}$
(d)	1	$g_{\tilde{1}}=(h_{\tilde{5}}H_2H_5H_{5'})^{-1}$ $g_{\tilde{3}}=h_{\tilde{5}}^{-1}(H_5H_{5'})^{-1}, g_{\tilde{4}}=h_{\tilde{5}}^{-1}H_5^{-1}$ $g_{\tilde{6}}=h_{\tilde{5}}^{-1}H_{5'}^{-1}, g_{\tilde{7}}=h_{\tilde{5}}^{-1}$	$g_2=H_2^{-1}$ $g_5=H_5^{-1}H_{5'}^{-1}$ $g_8=H_5^{-1}, g_9=H_{5'}^{-1}$
(f)	2	$g_{\tilde{1}}=h_{\tilde{5}}^{-1}H_2^{-1}H_5^{-1}$ $g_{\tilde{3}}=g_{\tilde{4}}=h_{\tilde{5}}^{-1}H_5^{-1}H_{5'}^{-1}$ $g_{\tilde{5}}=h_{\tilde{5}}^{-1}H_{5'}^{-1}, g_{\tilde{7}}=h_{\tilde{5}}^{-1}$	$g_2=H_2^{-1}H_{5'}^{-1}$ $g_6=H_5^{-1}H_{5'}^{-1}$ $g_8=H_5^{-1}$
(g)	2	$g_{\tilde{2}}=h_{\tilde{5}}^{-1}H_2^{-1}$ $g_{\tilde{3}}=g_{\tilde{4}}=h_{\tilde{5}}^{-1}H_5^{-1}H_{5'}^{-1}$ $g_{\tilde{5}}=h_{\tilde{5}}^{-1}H_5^{-1}, g_{\tilde{6}}=h_{\tilde{5}}^{-1}H_{5'}^{-1}$	$g_1=H_2^{-1}H_5^{-1}H_{5'}^{-1}$ $g_7=H_5^{-1}$ $g_8=H_{5'}^{-1}$

Table 13. (Continue) Concrete metrics for four M-branes I.

eqs. (C.4a), (C.4b), and (C.4d) are written by linear combinations of terms depending on both t and z^i, and those depending only on z^i. Then, in order to satisfy eqs. (C.4a), (C.4b), and (C.4d), we obtain

$$\partial_t^2 K_{\rm w}=0, \qquad \triangle_{\rm Z} H_{\rm w}=0. \tag{C.6}$$

We note that the function $K_{\rm w}(t)$ depends on the linear function of the time t. The Einstein

		0	1	2	3	4	5	6	7	8	9	10	$\tilde{I}$	cos	BH
$(\mathrm{M2})^2$ $(\mathrm{M5})^2$	M2	$\circ$	$\circ$							$\circ$			(a)	–	–
	M2	$\circ$		$\circ$							$\circ$				
	M5	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$						(b)	–	–
	M5	$\circ$	$\circ$	$\circ$	$\circ$			$\circ$	$\circ$						
	M2	$\circ$	$\circ$							$\circ$			(c)	–	–
	M2	$\circ$				$\circ$		$\circ$					(d)	–	–
	M5	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$								
	M5	$\circ$	$\circ$	$\circ$	$\circ$			$\circ$	$\circ$				(e)	–	–
	M2	$\circ$				$\circ$		$\circ$					(f)	$\surd$	$\surd$
	M2	$\circ$					$\circ$		$\circ$						
	M5	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$						(g)	$\surd$	$\surd$
	M5	$\circ$	$\circ$	$\circ$	$\circ$			$\circ$	$\circ$						
$(\mathrm{M2})^3$ M5	M2	$\circ$	$\circ$	$\circ$									(a)	–	–
	M2	$\circ$			$\circ$	$\circ$									
	M2	$\circ$					$\circ$	$\circ$							
	M5	$\circ$	$\circ$		$\circ$		$\circ$		$\circ$	$\circ$			(b)	–	–
$(\mathrm{M2})^4$	M2	$\circ$	$\circ$	$\circ$									(a)	–	–
	M2	$\circ$			$\circ$	$\circ$									
	M2	$\circ$					$\circ$	$\circ$							
	M2	$\circ$							$\circ$	$\circ$					

Table 14. Intersections of four M-branes II.

equations with the metric (C.3) are reduced to

$$\begin{aligned} h_{\mathrm{w}}(t,z) &= K_{\mathrm{w}}(t) + H_{\mathrm{w}}(z), \\ \partial_t^2 K_{\mathrm{w}} &= 0, \\ \triangle_{\mathrm{Z}} H_{\mathrm{w}} &= 0, \\ R_{ij}(\mathrm{Z}) &= 0. \end{aligned} \tag{C.7}$$

Especially, for the case of $u_{ij} = \delta_{ij}$, we find the solution of the KK wave as

$$\begin{aligned} ds^2 &= -dt^2 + d\zeta^2 + f_{\mathrm{w}}\,(dt - d\zeta)^2 + u_{ij} dz^i dz^j \\ f_{\mathrm{w}}(t,z) &\equiv h_{\mathrm{w}}(t,z) - 1, \end{aligned} \tag{C.8}$$

with

$$\begin{aligned} h_{\mathrm{w}}(t,z) &= K_{\mathrm{w}}(t) + H_{\mathrm{w}}(z), \\ K_{\mathrm{w}}(t) &= A_{\mathrm{w}} t + B_{\mathrm{w}} \\ H_{\mathrm{w}}(z) &= C_{\mathrm{w}} + \sum_k \frac{Q_{\mathrm{w},k}}{|\boldsymbol{z} - \boldsymbol{z}_k|^{D-4}}, \end{aligned} \tag{C.9}$$

where A_{w}, B_{w}, C_{w}, Q_{w} are constant parameters and $\boldsymbol{z}_k$ represent the positions of the branes in Z space.

C.2 KK-monopole

Next we discuss the dynamical solution of KK-monopole [35, 36]. In the reduced $(D-1)$-dimensional picture, it has to be a magnetically charged $(D-5)$-brane with a 2-form $F_{(D-3)}$.

$\mathrm{M2^2M5^2}$		$\mathcal{A}=(h_{\tilde{2}}H_2)^{1/3}(H_5H_{5'})^{2/3}$	$g_{\tilde{0}}=(h_{\tilde{2}}H_2H_5H_{5'})^{-1}$
	$\dim(Z)$	$g_{\tilde{\alpha}}$	g_{α}
(a)	1	$g_{\tilde{1}}=(h_{\tilde{2}}H_5H_{5'})^{-1}$ $g_8=h_{\tilde{2}}^{-1}$	$g_2=(H_2H_5H_{5'})^{-1}, g_3=(H_5H_{5'})^{-1},$ $g_4=g_5=H_5^{-1}, g_6=g_7=H_{5'}^{-1}$ $g_9=H_2^{-1}$
(c)	2	$g_{\tilde{1}}=(h_{\tilde{2}}H_5H_{5'})^{-1}$ $g_8=h_{\tilde{2}}^{-1}$	$g_2=g_3=(H_5H_{5'})^{-1}$ $g_4=(H_2H_5)^{-1}, g_5=H_5^{-1}$ $g_6=(H_2H_{5'})^{-1}, g_7=H_{5'}^{-1}$
(d)	2	$g_{\tilde{4}}=(h_{\tilde{2}}H_5H)^{-1}$ $g_{\tilde{6}}=(h_{\tilde{2}}H_{5'})^{-1},$	$g_1=(H_2H_5H_{5'})^{-1}$ $g_2=g_3=(H_5H_{5'})^{-1}$ $g_5=H_5^{-1}, g_7=H_{5'}^{-1}, g_8=H_2^{-1}$
(f)	3	$g_{\tilde{4}}=(h_{\tilde{2}}H_5)^{-1}$ $g_{\tilde{6}}=(h_{\tilde{2}}H_{5'})^{-1},$	$g_1=g_2=g_3=(H_5H_{5'})^{-1}$ $g_5=(H_2H_5)^{-1}, g_7=(H_2H_{5'})^{-1}$
$\mathrm{M2^2M5^2}$		$\mathcal{A}=h_{\tilde{5}}^{2/3}(H_2H_{2'})^{1/3}H_5^{2/3}$	$g_0=(h_{\tilde{5}}H_2H_{2'}H_5)^{-1}$
	$\dim(Z)$	$g_{\tilde{\alpha}}$	g_{α}
(b)	1	$g_{\tilde{1}}=(h_{\tilde{5}}H_2H_5)^{-1}$ $g_{\tilde{2}}=(h_{\tilde{5}}H_{2'}H_5)^{-1}$ $g_{\tilde{3}}=(h_{\tilde{5}}H_5)^{-1}, g_{\tilde{4}}=g_{\tilde{5}}=h_{\tilde{5}}^{-1}$	$g_6=g_7=H_5^{-1}$ $g_8=H_2^{-1}$ $g_9=H_{2'}^{-1}$
(e)	2	$g_{\tilde{1}}=(h_{\tilde{5}}H_2H_5)^{-1}$ $g_{\tilde{2}}=g_{\tilde{3}}=(h_{\tilde{5}}H_5)^{-1}$ $g_{\tilde{4}}=(h_{\tilde{5}}H_{2'})^{-1}, g_{\tilde{5}}=h_{\tilde{5}}^{-1}$	$g_6=(H_2H_5)^{-1}$ $g_7=H_5^{-1}$ $g_8=H_2^{-1}$
(g)	3	$g_{\tilde{1}}=g_{\tilde{2}}=g_{\tilde{3}}=(h_{\tilde{5}}H_5)^{-1}$ $g_{\tilde{4}}=(h_{\tilde{5}}H_2)^{-1}, g_{\tilde{5}}=(h_{\tilde{5}}H_{2'})^{-1}$	$g_6=(H_2H_5)^{-1}$ $g_7=(H_{2'}H_5)^{-1}$
$\mathrm{M2^3M5}$		$\mathcal{A}=h_{\tilde{2}}^{1/3}(H_2H_{2'}H_5)^{2/3}$	$g_0=(h_{\tilde{2}}H_2H_{2'}H_5)^{-1}$
	$\dim(Z)$	$g_{\tilde{\alpha}}$	g_{α}
(a)	2	$g_{\tilde{1}}=(h_{\tilde{2}}H_5)^{-1}$ $g_{\tilde{2}}=h_{\tilde{2}}^{-1}$	$g_3=(H_2H_5)^{-1}, g_4=H_2^{-1}$ $g_5=(H_{2'}H_5)^{-1}, g_6=H_{2'}^{-1}$ $g_7=g_8=H_5^{-1}$
$\mathrm{M2^3M5}$		$\mathcal{A}=h_{\tilde{5}}^{2/3}(H_2H_{2'}H_{2''})^{1/3}$	$g_0=(h_{\tilde{5}}H_2H_{2'}H_{2''})^{-1}$
	$\dim(Z)$	$g_{\tilde{\alpha}}$	g_{α}
(b)	2	$g_{\tilde{1}}=(h_{\tilde{5}}H_2)^{-1}, g_{\tilde{3}}=(h_{\tilde{5}}H_{2'})^{-1}$ $g_{\tilde{5}}=(h_{\tilde{5}}H_{2''})^{-1}$ $g_{\tilde{7}}=g_{\tilde{8}}=h_{\tilde{5}}^{-1}$	$g_2=H_2^{-1}$ $g_4=H_{2'}^{-1}$ $g_6=H_{2''}^{-1}$
$\mathrm{M2^4}$		$\mathcal{A}=(h_{\tilde{2}}H_2H_{2'}H_{2''})^{2/3}$	$g_0=(h_{\tilde{2}}H_2H_{2'}H_{2''})^{-1}$
	$\dim(Z)$	$g_{\tilde{\alpha}}$	g_{α}
(a)	2	$g_{\tilde{1}}=g_{\tilde{2}}=h_{\tilde{2}}^{-1}$	$g_3=g_4=H_2^{-1}$ $g_5=g_6=H_{2'}^{-1}$ $g_7=g_8=H_{2''}^{-1}$

Table 15. (Continue) Concrete metrics for four M-branes II.

Replacing D with $(D-1)$ and substituting $p=(D-5)$ into eqs. (2.3a), (2.3b), (2.3c), (2.6), and (A.1), the electric $(D-5)$-brane solution in $(D-1)$ dimensions can be written as

$$\begin{aligned}
ds^2_{(D-1)} &= h_{\mathrm{m}}^{-\frac{1}{D-3}}q_{\mu\nu}dx^\mu dx^\nu + h_{\mathrm{m}}^{\frac{D-4}{D-3}}u_{ij}(\mathrm{Z})dz^i dz^j, \\
e^\phi &= h_{\mathrm{m}}^{\sqrt{\frac{(D-2)}{2(D-3)}}}, \qquad\qquad F_{(D-3)} = d(h_{\mathrm{m}}^{-1})\wedge\sqrt{-q}dx^0\wedge\cdots\wedge dx^{D-5}, \\
R_{\mu\nu}(\mathrm{X}) &= 0, \quad R_{ij}(\mathrm{Z})=0, \\
h_{\mathrm{m}}(x,z) &= K_{\mathrm{m}}(x)+H_{\mathrm{m}}(z), \quad D_\mu D_\nu K_{\mathrm{m}}=0, \triangle_{\mathrm{Z}}H_{\mathrm{m}}=0,
\end{aligned} \tag{C.10}$$

where $R_{\mu\nu}(\mathrm{X})$, D_μ and q are Ricci tensor, covariant derivative, determinant constructed from the $(D-4)$-dimensional metric $q_{\mu\nu}$ which depends only on the coordinate x^μ, $R_{ij}(\mathrm{Z})$

branes	dim(Z)	1M	2M	3M	4M	5M	$\tilde{I}(\#)$	cos	BH
$(M5)^5$	1	2	2	2	2	1	M5(3)	–	–
	1	1	4	2	0	2	M5(2)	–	–
	2	0	2	4	1	1	M5(2)	–	–
$M2(M5)^4$	1	3	1	3	2	0	M2(1)	–	–
							M5(2)	–	–
	2	0	4	2	2	0	M2(1)	–	–
							M5(1)	–	–
	1	1	6	0	1	1	M2(1)	–	–
							M5(1)	–	–
	1	2	3	3	0	1	M2(1)	–	–
							M5(2)	–	–
$(M2)^2(M5)^3$	1	3	3	2	1	0	M2(2)	–	
							M5(2)	–	–
	2	1	3	4	0	0	M2(1)	–	–
							M5(2)	–	–
$(M2)^3(M5)^2$	1	4	3	2	0	0	M2(2)	–	–
							M5(1)	–	–
	2	1	6	1	0	0	M2(2)	–	–
							M5(1)	–	–
$(M2)^4(M5)^1$	1	5	4	0	0	0	M2(1)	–	–
							M5(1)	–	–

Table 16. Intersections of five M-branes.

branes	dim(Z)	1M	2M	3M	4M	5M	6M	$\tilde{I}(\#)$	cos	BH
$(M5)^6$	1	0	3	4	0	0	2	M5(1)	–	–
	1	1	2	2	2	1	1	M5(3)	–	–
	2	0	0	4	3	0	1	M5(1)	–	–
$M2(M5)^5$	1	1	2	4	1	0	1	M2(1)	–	–
								M5(2)	–	–
$(M2)^2(M5)^4$	1	1	4	2	1	1	0	M2(2)	–	–
								M5(1)	–	–
	1	2	2	2	3	0	0	M2(1)	–	–
								M5(2)	–	–
	2	0	2	4	2	0	0	M2(1)	–	–
								M5(1)	–	–
$(M2)^3(M5)^3$	1	2	3	3	1	0	0	M2(2)	–	–
								M5(2)	–	–
	2	0	3	5	0	0	0	M2(1)	–	–
								M5(1)	–	–
$(M2)^4(M5)^2$	1	2	5	2	0	0	0	M2(2)	–	–
								M5(1)	–	–

Table 17. Intersections of six M-branes.

and $\triangle_{\rm Z}$ are Ricci tensor, Laplace operator with respect to the three-dimensional metric u_{ij} which depends only on the coordinate z^i. Before going to D dimensions, we have to rescale the metric (C.10) to put it in the D-dimensional Einstein frame. Then we use the conformal transformation

$$\bar{g}_{MN} = h_{\rm m}^{-1/(D-3)} g_{MN}. \tag{C.11}$$

Collecting the above results, we find the D-dimensional metric of KK-monopole:

$$ds^2 = g_{MN} dx^M dx^N$$

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branes	dim(Z)	1M	2M	3M	4M	5M	6M	7M	$\tilde{I}(\#)$	cos	BH
$(\mathrm{M5})^7$	1	1	0	4	0	3	0	1	M5(2)	–	–
	1	0	3	0	4	0	1	1	M5(2)	–	–
	1	0	0	7	0	0	0	2	M5(2)	–	–
	2	0	0	0	7	0	0	1	M5(2)	–	–
$\mathrm{M2(M5)}^6$	1	1	0	4	3	0	0	1	M2(1)	–	–
									M5(1)	–	–
$(\mathrm{M2})^3(\mathrm{M5})^4$	1	1	2	4	1	1	0	0	M2(2)	–	–
									M5(1)	–	–
	1	1	3	1	4	0	0	0	M2(1)	–	–
									M5(2)	–	–
	2	0	0	6	2	0	0	0	M2(1)	–	–
									M5(1)	–	–
$(\mathrm{M2})^4(\mathrm{M5})^3$	1	1	3	4	1	0	0	0	M2(2)	–	–
									M5(1)	–	–

Table 18. Intersections of seven M-branes.

branes	dim(Z)	1M	2M	3M	4M	5M	6M	7M	8M	$\tilde{I}(\#)$	cos	BH
$\mathrm{M2(M5)}^7$	1	1	0	0	7	0	0	0	1	M2(1)	–	–
										M5(1)	–	–
$(\mathrm{M2})^4(\mathrm{M5})^4$	1	0	4	0	5	0	0	0	0	M2(1)	–	–
										M5(1)	–	–
	1	1	0	6	1	1	0	0	0	M2(2)	–	–
										M5(1)	–	–

Table 19. Intersections of eight M-branes.

$$= q_{\mu\nu}(\mathrm{X})dx^\mu dx^\nu + h_{\mathrm{m}}^{-1}\left(d\zeta + \mathcal{A}_i^{(\mathrm{m})}dz^i\right)^2 + h_{\mathrm{m}} u_{ij}(\mathrm{Z})dz^i dz^j\,, \tag{C.12}$$

where $q_{\mu\nu}$ is $(D-4)$-dimensional metric depends only on the coordinate x^μ, and u_{ij} denotes the three-dimensional metric depends only on the transverse coordinate z^i, and the function h_{m} depends on x^μ as well as z^i, and relation between h_{m} and $\mathcal{A}_i^{(\mathrm{m})}$ is

$$\mathcal{F}_{ij} \equiv \partial_i \mathcal{A}_j^{(\mathrm{m})} - \partial_j \mathcal{A}_i^{(\mathrm{m})} = -\epsilon_{ijk}\,\partial^k h_{\mathrm{m}}. \tag{C.13}$$

Substituting the metric (C.12) into the D-dimensional vacuum Einstein equations, we obtain

$$R_{\mu\nu}(\mathrm{X}) - h_{\mathrm{m}}^{-1} D_\mu D_\nu h_{\mathrm{m}} = 0, \tag{C.14a}$$
$$h_{\mathrm{m}}^{-1}\partial_\mu \partial_i h_{\mathrm{m}} = 0, \tag{C.14b}$$
$$h_{\mathrm{m}}^{-2}\triangle_{\mathrm{X}} h_{\mathrm{m}} = 0, \tag{C.14c}$$
$$\mathcal{A}_i^{(\mathrm{m})} h_{\mathrm{m}}^{-2}\triangle_{\mathrm{X}} h + \mathcal{A}_i^{(\mathrm{m})} h_{\mathrm{m}}^{-3}\triangle_{\mathrm{Z}} h_{\mathrm{m}} = 0, \tag{C.14d}$$
$$R_{ij}(\mathrm{Z}) - \frac{1}{2}\left(u_{ij} - h_{\mathrm{m}}^{-2}\mathcal{A}_i^{(\mathrm{m})}\mathcal{A}_j^{(\mathrm{m})}\right)\triangle_{\mathrm{X}} h_{\mathrm{m}} - \frac{1}{2} h_{\mathrm{m}}^{-1} u_{ij}\triangle_{\mathrm{Z}} h_{\mathrm{m}} = 0, \tag{C.14e}$$

where D_μ, $\triangle_{\mathrm{X}}$, $R_{\mu\nu}(\mathrm{X})$ are covariant derivative, Laplace operator, Ricci tensor with respect to the metric $q_{\mu\nu}$, and $\triangle_{\mathrm{Z}}$, $R_{ij}(\mathrm{Z})$ are Laplace operator, Ricci tensor with respect to the metric u_{ij}, and we assume

$$\partial_\mu \mathcal{A}_i^{(\mathrm{m})} = 0. \tag{C.15}$$

Using eq. (C.14b), we get

$$h_{\mathrm{m}}(x,z) = K_{\mathrm{m}}(x) + H_{\mathrm{m}}(z). \tag{C.16}$$

Equations (C.14a), (C.14d), (C.14e) are written by the combination of the term depending not only on x^α but z^i, and depending only on z^i. Then, in order to satisfy eqs. (C.14a), (C.14b), and (C.14c), we choose

$$D_\mu D_\nu K = 0, \qquad \triangle_{\mathrm{Z}} H = 0, \qquad R_{\alpha\beta}(\mathrm{X}) = 0, \qquad R_{ij}(\mathrm{Z}) = 0. \tag{C.17}$$

The Einstein equations in the metric (C.12) are then reduced to

$$\begin{aligned} h_{\mathrm{m}}(x,z) &= K_{\mathrm{m}}(x) + H_{\mathrm{m}}(z), \\ D_\mu D_\nu K_{\mathrm{m}} &= 0, \\ \triangle_{\mathrm{Z}} H_{\mathrm{m}} &= 0, \\ R_{\mu\nu}(\mathrm{X}) &= 0, \quad R_{ij}(\mathrm{Z}) = 0. \end{aligned} \tag{C.18}$$

For $q_{\mu\nu} = \eta_{\mu\nu}$, $u_{ij} = \delta_{ij}$, we can obtain the solution of Einstein equation explicitly

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu + h_{\mathrm{m}} (dz^i)^2 + h_{\mathrm{m}}^{-1} \left(d\zeta + \mathcal{A}_i^{(\mathrm{m})} dz^i \right)^2, \tag{C.19}$$

with

$$\begin{aligned} h_{\mathrm{m}}(x,z) &= K_{\mathrm{m}}(x) + H_{\mathrm{m}}(z), \\ K_{\mathrm{m}}(x) &= A_{\mathrm{m}\,(\mu)}\, x^\mu + B_{\mathrm{m}}, \\ H_{\mathrm{m}}(z) &= C_{\mathrm{m}} + \sum_k \frac{Q_{\mathrm{m},\,k}}{|\boldsymbol{z} - \boldsymbol{z}_k|}, \end{aligned} \tag{C.20}$$

where $A_{\mathrm{m}\,(\mu)}$, B_{m}, C_{m}, $Q_{\mathrm{m},\,k}$, $\boldsymbol{z}_k$'s are integration constants.

D Intersecting branes with M-waves and KK-monopoles

A complete list for static brane system with M-waves and KK-monopoles are given in [34]. Hence we pick up only interesting cases in which one can discuss cosmology or a black hole (object) in table 20. In the table, circles indicate where the brane world-volumes enter, ζ represents the coordinate of the KK-monopole, and the time-dependent branes are indicated by (a) and (b) and so on for different solutions. When the solutions can be used for cosmology and black hole physics, they are marked in the corresponding columns. The applications of these solutions to cosmology and black hole physics are discussed in section 5.

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		0	1	2	3	4	5	6	7	8	9	10	$\tilde{I}$	cos	BH
M2-M2-KKM	M2	$\circ$	$\circ$	$\circ$									(a)	–	$\surd$
	M2	$\circ$			$\circ$	$\circ$									
	KKM	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	ζ	$\mathcal{A}_8^{(\mathrm{m})}$	$\mathcal{A}_9^{(\mathrm{m})}$	$\mathcal{A}_{10}^{(\mathrm{m})}$	(b)	–	$\surd$
	M2	$\circ$	$\circ$	$\circ$									(c)	$\surd$	–
	M2	$\circ$			$\circ$	$\circ$							(d)	$\surd$	–
	KKM	$\circ$	ζ	$\mathcal{A}_2^{(\mathrm{m})}$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\mathcal{A}_9^{(\mathrm{m})}$	$\mathcal{A}_{10}^{(\mathrm{m})}$	(e)	$\surd$	–
M2-M5-W	M2	$\circ$	$\circ$	$\circ$									(a)	$\surd$	$\surd$
	M5	$\circ$	$\circ$		$\circ$	$\circ$	$\circ$	$\circ$					(b)	$\surd$	$\surd$
	W	$\circ$	ζ										(c)	$\surd$	$\surd$
M2-M5-KKM	M2	$\circ$	$\circ$	$\circ$									(a)	$\surd$	$\surd$
	M5	$\circ$	$\circ$		$\circ$	$\circ$	$\circ$	$\circ$					(b)	$\surd$	$\surd$
	KKM	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	ζ	$\mathcal{A}_8^{(\mathrm{m})}$	$\mathcal{A}_9^{(\mathrm{m})}$	$\mathcal{A}_{10}^{(\mathrm{m})}$	(c)	$\surd$	$\surd$
	M2	$\circ$	$\circ$	$\circ$									(d)-1, 2	$\surd$	–
	M5	$\circ$	$\circ$		$\circ$	$\circ$	$\circ$	$\circ$					(e)-1, 2	$\surd$	–
	KKM	$\circ$	ζ	$\mathcal{A}_2^{(\mathrm{m})}$	$\circ$	$\circ$	$\circ$	$\mathcal{A}_6^{(\mathrm{m})}$	$\circ$	$\circ$	$\circ$	$\mathcal{A}_{10}^{(\mathrm{m})}$	(f)-1, 2	$\surd$	–
M2-M5-W-KKM	M2	$\circ$	$\circ$	$\circ$									(a)	$\surd$	$\surd$
	M5	$\circ$	$\circ$		$\circ$	$\circ$	$\circ$	$\circ$					(b)	$\surd$	$\surd$
	W	$\circ$	ζ^1										(c)	$\surd$	$\surd$
	KKM	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	ζ^7	$\mathcal{A}_8^{(\mathrm{m})}$	$\mathcal{A}_9^{(\mathrm{m})}$	$\mathcal{A}_{10}^{(\mathrm{m})}$	(d)	$\surd$	$\surd$
M5-M5-W	M5	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$						(a)	–	$\surd$
	M5	$\circ$	$\circ$	$\circ$	$\circ$			$\circ$	$\circ$						
	W	$\circ$	ζ										(b)	–	$\surd$
M5-M5-KKM	M5	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$						(a)	$\surd$	–
	M5	$\circ$	$\circ$	$\circ$	$\circ$			$\circ$	$\circ$				(b)	$\surd$	–
	KKM	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	ζ	$\mathcal{A}_7^{(\mathrm{m})}$	$\circ$	$\mathcal{A}_9^{(\mathrm{m})}$	$\mathcal{A}_{10}^{(\mathrm{m})}$	(c)	$\surd$	–
M5-M5-KKM-KKM	M5	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$						(a)	$\surd$	–
	M5	$\circ$	$\circ$	$\circ$	$\circ$			$\circ$	$\circ$						
	KKM	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	ζ^6	$\mathcal{A}_7^{(\mathrm{m})}$	$\circ$	$\mathcal{A}_9^{(\mathrm{m})}$	$\mathcal{A}_{10}^{(\mathrm{m})}$	(b)	$\surd$	–
	KKM	$\circ$	$\circ$	$\circ$	$\circ$	ζ^4	$\mathcal{B}_5^{(\mathrm{m})}$	$\circ$	$\circ$	$\circ$	$\mathcal{B}_9^{(\mathrm{m})}$	$\mathcal{B}_{10}^{(\mathrm{m})}$			

Table 20. Intersecting M-branes with M-wave and KK-monopole. Here we show only the interesting cases which can be applied to cosmology or a black hole system. The labelling (a), (b), $\cdots$ in the column "$\tilde{I}$" denotes which brane (or wave, KK-monopole) is time dependent. In the second case of M2-M5-KKM system, there are two possibilities which space dimensions can be our three space, i.e., the case 1: $[(\xi^1, \xi^2, \xi^3) = (x^3, x^4, x^5)]$ and the case 2: $[(\xi^1, \xi^2, \xi^3) = (x^7, x^8, x^9)]$. We show them by (d)-1 (d)-2, (e)-1, (e)-2, or (f)-1, (f)-2.

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